Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

L. H. Donnell, *Editor*

T. von Kármán, S. Timoshenko, *Editorial Advisers*

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Abbreviations of units follow the standard of Abbreviations for Scientific and Engineering Terms of the Am. Standards Assoc. Examples: psi (pounds per square inch); cps (cycles per second); mph (miles per hour).

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What's in a Name?

MOST of us have had the experience of searching for a word to express some thing or idea, and of finding that the language has a blank spot—that there is no single word which serves the purpose. No language, however rich, could have special words to express every thing or idea with which men concern themselves, but there are blank spots for some surprisingly important concepts. Consider the fact that, while there are special names for citizens of almost every country—Mexicans, Poles, Chinese, Englishmen—there is no special name for a citizen of the United States. The unfortunate people of this country cannot call themselves Unitedstatesers or Usans, and they use the word American at the risk of confusion or resentment for appropriating a name to which others have equal rights.

The United States no doubt gets along well enough in spite of this handicap! This discussion is inspired by the difficulty which recurrently confronts the writer in describing workers in the field of Mechanics, and the language gap here may cause a much more serious handicap. It is not improbable that public recognition, and growth of professional pride and fellowship, come much more easily to groups which are blessed with simple, unequivocal labels, such as surgeons, lawyers, physicists.

It may be questioned whether it is possible or desirable to promote group consciousness or recognition as a group for as diverse a crowd as practitioners in Mechanics. Most of us consider ourselves members of other groups—mechanical, civil, or aeronautical engineers, mathematicians, physicists, metallurgists, teachers, etc. And yet, for the growing group of scientists who—whatever they started from—have come to devote their lives in one way or another to the manifold problems arising from the interaction between force and matter, such designations have become misleading and unsatisfactory. Those of us who have filled out the questionnaire of the National Roster of Scientific Personnel have had brought home to us both the difficulty of describing our activities adequately without using the word "Mechanics," and the lack of recognition which our anonymity has brought to us.

One way of helping to correct this situation is to decide on and to use, at least among ourselves, some reasonable and suitable designation for our profession. If we try to think of such a name we are at once reminded that we have a double handicap—not only do we have no name for workers in our field, but the established name for the field itself is one which is associated in the public's mind with entirely different activities. It is not merely the uneducated to whom the word Mechanics brings a picture of a man in overalls, probably greasy, wielding a hammer or a wrench (not that we have any quarrel with our friends of the overalls—many of us are proud to boast that we are good mechanics as well as practitioners of Mechanics).

"Mechanology" is a recognized word for the science of Mechanics. It is at least as suitable a name for our field as the more common one—and would certainly be far less easily misunderstood. While the name Mechanics is too well established, both in English and in related forms in all languages, to be displaced in the near future, it might be desirable to make a practice of using the alternative word Mechanology whenever feasible. And its natural derivative, "mechanologist," seems to be the obvious word for practitioners in our field; it is more logical and, like its root, much less easily misinterpreted than such words as "mechanist," "mechanicist," etc.

Comments are invited, not only on the choice of a name, but on the broader idea of the desirability of attempting to establish our field of activity as a separate profession. In the meantime we propose, until something better is suggested, to use the word "mechanologist" to describe most of the contributors and readers of this magazine.

L. H. Donnell

General Theoretical and Experimental Methods

(See also Revs. 769, 770, 813, 908)

758. G. **Kind-Schaad, "Solution of eigenvalue problems by means of punched-card tabulating machines (Lösung von Eigenwertproblemen mittels Lochkartenmaschinen),"** *Schweiz. Arch.*, June 1947, vol. 13, pp. 161–168.

The use of International Business Machines of the punched-card type is described. The problems discussed are those in which the number of degrees of freedom is finite, such as the calculation of the modes of vibration of a mechanical system, the deformation of an elastic structure, or the conditions for the stability of a body under stress. Although the simpler problems of these types can usually be solved by numerical methods, the machine method allows problems having many degrees of freedom to be solved easily.

An iteration process is used in order to make use of machines capable of multiplying two numbers or adding a group of numbers. A trial value of the dominant root is guessed and is improved successively. The convergence is most rapid for characteristic frequencies or "eigenwerte" which are widely different in absolute value; the process must be repeated for each additional frequency. Both real and imaginary roots of the characteristic equation may be found. Cases of repeated roots cannot be handled.

The method described is an adaptation for machines of the iterative procedure described by Frazer, Duncan, and Collar ["Elementary matrices," Cambridge, 1938]. Three different machines are used for sorting the cards, multiplying, and tabulating.

R. L. Pigford, USA

759. E. A. Goldberg and G. W. Brown, "An electronic simultaneous equation solver," *J. appl. Phys.*, Apr. 1948, vol. 19, pp. 339–345.

An electronic device capable of solving ten simultaneous linear algebraic equations, which has been built at the RCA Laboratories, is described. The coefficients and constant terms in the

equations to be solved are set on potentiometers interconnecting a number of high-gain amplifiers, and the unknown variables are read as voltages from a bridge having a cathode-ray tube indicator.

The basic block diagram of the equation solver is shown, and the stability and the accuracy of the system are discussed.

Stephen H. Crandall, USA

760. Willis A. Adcock, "An automatic simultaneous equation computer and its use in solving secular equations," *Rev. sci. Instrum.*, Mar. 1948, vol. 19, pp. 181–187.

This paper describes an automatic computer for solving linear simultaneous equations and secular equations, and describes the performance obtained from a four-equation model.

The coefficients of the equations are represented by settings on voltage dividers and the variables are represented by voltages. The method of solving these equations is an improvement on the Gauss-Siedel iteration method, in that all variables are assumed initially and the error in each equation activates a feed-back system which automatically adjusts the variables to the proper values.

As an example, three simultaneous equations are solved to an accuracy of less than 1 per cent in 15 sec, while a secular equation with four roots is solved to an accuracy of 1 per cent in a few minutes after setting the coefficients. Henry J. Barten, USA

761. L. A. Lusternik and A. M. Prokhorov, "The determination of eigenvalues and eigenfunctions of certain operators by means of a recurrent circuit" (in English), *C. R. Acad. Sci. URSS*, Mar. 10, 1947, vol. 55, pp. 575–578.

In this paper a method is given for finding the eigenvalues and eigenvectors of Sturm-Liouville and Fredholm symmetric operators by means of an electric circuit. The technique is to approximate the operators by a symmetric $n \times n$ matrix $\mathbf{A}$ which can be represented electrically as a recurrent network of resistors and capacitors. The key to the method is the fact that if the matrix $\mathbf{A}$ has eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$, and associated eigenvectors $y_1, y_2, \ldots, y_n$, then the solution of the vector differential equation $\mathbf{A}y = \mathbf{K}\,dy/dt$, $y(t) = \sum_{i=1}^{n} c_i y_i \exp(-\lambda_i t/\mathbf{K})$, is, for large t, given approximately by $c_1 y_1 \exp(-\lambda_1 t/\mathbf{K})$.

To find λ_1 the nodes of the network are excited by short pulses whose period is substantially longer than the time constant $1/\lambda_1$. The function $y/(t)$ can be viewed on the screen of a cathode-ray oscillograph and, by introducing suitable time signals $1/\lambda_1$, can be measured to an accuracy of between 3 and 5 per cent. To find λ_2 a similar procedure is followed except that the initial values of the impressed voltages at the nodes of the network are so chosen that they form the components of a vector orthogonal to the eigenvector y_1. The method can be extended to determine the other eigenvalues and eigenvectors. Benjamin Epstein, USA

General Dynamics, Kinematics, Friction

(**See also Revs. 765, 775, 827, 906, 915**)

762. Stanley U. Benscoter, "Effect of partial wing lift in seaplane landing impact," *Nat. adv. Comm. Aero. tech. Note*, no. 1563, Apr. 1948, pp. 1–14.

Equations of motion are developed for a prismatic float of infinite length with partial wing lift entering a fluid. The solution is similar in form to that derived when full wing lift is assumed. The solution shows that partial wing lift may have a certain small effect upon the maximum acceleration. The effect is small enough, however, to be disregarded in most practical design cases. The paper extends the general review of the seaplane landing problem given by the author in a previous paper ["Impact theory for seaplane landings," *Nat. adv. Comm. Aero. tech. Note*, no. 1437, Oct. 1947]. Samuel Levy, USA

763. André Chartet, "General properties of rolling contacts. Similitude theory (Propriétes generales des contacts de roulement. Théorie des similitudes)," *C. R. Acad. Sci. Paris*, Nov. 24, 1947, vol. 225, pp. 986–988.

The author considers two elastic bodies of revolution which are pressed together by a given force. These bodies can rotate about axes which are assumed to be initially in the same plane. He then establishes laws of rolling friction, on the basis of certain theoretical considerations, and of considerations of "similitude" of the contacts. The latter considerations demand eleven conditions, of which only three are given.

The author gives only final results, without going into the reasoning behind them. He states that his results are confirmed by experiments, which will be described in a later paper.

Ratip Berker, Turkey

764. Karl Pflanz, "Axle loading and velocity on curved tracks (Achsdruck und Fahrgeschwindigkeit in Gleisbogen)," *Schweiz. Bauztg.*, 1947, vol. 65, Nov. 8, pp. 611–614; Nov. 15, pp. 623–627.

The author considers a locomotive with B_0-B_0 (Austrian) arrangement of axles and 14 to 20 metric tons axle loading on circular tracks of 250, 400, and 625 meters radius. The author sets himself the task of determining the effect on the forces between rails and wheels of an increase in speed of 10 km per hr above the speeds normal for the above curves (60, 80, and 100 km per hr respectively). He concludes that the static forces, that is, the forces due to rim friction and that portion of the centrifugal force not compensated by the inclination of the track, are little affected. The increase of speed, however, materially affects the dynamic forces due to turning of the locomotive, caused by deviations of the track from true curvature.

F. Hymans, USA

Gyroscopics, Governors, Servomechanisms

(**See also Revs. 770, 771**)

765. A. I. Lourye, "On the stability of motion of a dynamic system" (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, July-Aug. 1947, vol. 11, pp. 445–448.

The author investigates the conditions of stability of a discontinuously controlled dynamic system. The differential equation of the system in terms of dimensionless parameters is $\ddot{\zeta} + 2n\dot{\zeta} + \zeta = \mu$ where ζ is a parameter to be controlled and μ is the controlling parameter. The controlling system is arranged so that means registering ζ and $\dot{\zeta}$ are combined according to the relation $\sigma = \zeta + \lambda\dot{\zeta}$, so that σ may be regarded as an additional parameter.

The servomotor system operates discontinuously, so that when σ and $\dot{\sigma}$ are of opposite signs, the control action is absent; when they are of the same sign, this action is exerted at a constant rate in one direction or the other, according to the sign of σ and $\dot{\sigma}$. This results in the equation of control $\dot{\mu} = -1/2$ (sgn σ + sgn $\dot{\sigma}$) where the function $f(x) = \text{sgn}\, x$ is $+1$ for $x > 0$ and -1 for $x < 0$; $f(0) = 0$. As was shown previously by the author, the above

system of equations can be reduced to the canonical form by introducing certain new variables

$$\dot{x}_1 = -\rho_1 x_1 - \dot{\mu} \ ; \ \dot{x}_2 = -\rho_2 x_2 - \dot{\mu} \ ; \ \dot{\sigma} = \beta_1 x_1 + \beta_2 x_2 \ldots \text{[1.8]}$$

x_1 and x_2 being the new variables and ρ_1, ρ_2 the roots of an auxiliary equation.

The rest of the paper is devoted to the investigation of the stability of the system by the method of Liapounoff, which consists of introducing an appropriate quadratic form V of new variables, regarding the sign of which one can make definite conclusions. The author shows that by a proper choice of functions V (one for the real, and the other for the complex conjugate ρ_1 and ρ_2) the system [1.8] can be replaced by a single equation

$$\dot{V} = \frac{\partial V}{\partial x_1}\dot{x}_1 + \frac{\partial V}{\partial x_2}\dot{x}_2 + \frac{\partial V}{\partial \sigma}\dot{\sigma} = \phi(x_1, x_2, \sigma; \ \rho_1, \rho_2; \ A, a_1, a_2)$$

where A, a_1, and a_2 are positive real constants, the latter two being arbitrary; by imposing additional relations on a_1 and a_2, the right-hand side is negative, so that the motion is stable (provided $\lambda n > 1/2$) for any arbitrary initial conditions.

N. Minorsky, USA

Vibrations, Balancing

(See also Revs. 774, 778, 827, 884, 893)

766. A. I. Gubanov, "Vibrations of solid bodies in an elastic viscous medium" (in Russian), *J. tech. Phys.* (*Zh. tekh. Fiz.*), 1947, vol. 17, no. 5, pp. 525–536.

The author considers a continuous medium possessing both viscosity and rigidity. The equations of motion for the medium which are used by the author are due to J. I. Frenkel ["Kinetic theory of liquids," Moscow, 1945]. They are

$$\mu \nabla^2 \mathbf{v} = \rho \left(1 + \tau \frac{\partial}{\partial t}\right) \frac{\partial \mathbf{v}}{\partial t} + \left(1 + \tau \frac{\partial}{\partial t}\right) \nabla p$$

where $\tau = \mu/G$, μ being the viscosity and G the shear modulus, and the other symbols have their customary significance. The boundary conditions are the same as those for a viscous fluid. By assuming $\mathbf{v} = \mathbf{v}_0 \exp(i\omega t)$, the author is able to solve several problems: (1) Fluid bounded on one side by an oscillating plane, on the other by a fixed plane; (2) fluid bounded by an oscillating disk; (3) by concentric cylinders, one fixed, one oscillating. He then discusses experiments based on such solutions which could be used to verify the theory, or, if this is accepted, to evaluate μ and G.

J. V. Wehausen, USA

767. J. C. Houbolt and R. A. Anderson, "Calculation of uncoupled modes and frequencies in bending or torsion of nonuniform beams," *Nat. adv. Comm. Aero. tech. Note*, no. 1522, Feb. 1948, pp. 1–75.

Calculations of the natural frequencies and modes of nonuniform beams in uncoupled bending and torsional vibration are made, based on the principle of the Stodola method. The author's procedure consists in solving the usual differential equations for the bending and torsional vibration of a beam by a method of successive approximations; the integrations involved are performed by improved numerical methods. The higher modes are found by use of the orthogonality property of the normal modes.

Nine examples, drawn from the field of aircraft structures, are given, illustrating the method of dealing with various boundary conditions. Comparison is made with available exact analytical solutions and the agreement is found to be good.

John L. Bogdanoff, USA

768. F. H. Todd and W. J. Marwood, "Ship vibration," *Trans. N. E. Coast Instn. Engrs. Shipb.*, Mar. 1948, vol. 64, pp. 193–210.

The natural frequency of vertical or horizontal vibration of a ship may be computed by considering it to be a free-free beam. It is found that the stiffness may be considered to be uniformly distributed, but that the correct nonuniform distribution of the mass must be considered. The inertia effect of the surrounding water must be included. The feature of greatest uncertainty appears to be the contribution of the superstructure to the stiffness of the ship.

Properties of 13 cargo and passenger ships are given, with measured and computed frequencies. Use of the stiffness at the midship section results in appreciable discrepancy between theory and experiment when the superstructures (decks) are short, but good agreement with long decks. A large number of examples are given for two-node vertical vibration. A few examples are given for two-node horizontal vibration and for three-node vertical vibration.

An approximate formula is given for estimating a ship frequency. Allowance is made for superstructures of various lengths. Factors in the formula are obtained from graphs prepared by plotting known quantities for a large number of ships.

Stanley U. Benscoter, USA

769. A. A. Dorodnitsin, "Asymptotic solution of Van der Pol's equation" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), May-June 1947, vol. 11, pp. 313–328.

Van der Pol's differential equation $x'' - \nu(1 - x^2)x' + x = 0$ is studied for large values of the parameter ν.

The entire cycle of oscillations in the phase plane x, x' is subdivided into four parts which belong to four overlapping domains, and the solutions are presented in the form of asymptotic power series in $1/\nu$ and $1/\nu^{2/3}$. Every one of the four parts of the cycle is represented by such an asymptotic series. The solutions thus obtained for different domains are connected, and general approximate expressions for the amplitude and the period of steady oscillations are given.

Since this question has been studied by others, and approximate expressions different from those of the author have been proposed, this reviewer has computed the frequency $f = 2\pi/T$ for $\nu = 1, 2, 5.6, 8, 10, 11.4$ with the aid of the author's final formula (8.12) and compared the approximate values of f with those found by Van der Pol and Shohat [*J. appl. Phys.*, 1944, vol. 15, p. 568]. The result is as follows:

Value of ν		1	2	5.6	8	10	11.4
Frequency according to	Van der Pol	0.90	0.78	0.38	0.39	0.31	0.24
	Shohat	0.93	0.77	0.43	0.35	0.30	0.25
	Author	0.73	0.79	0.51	0.40	0.33	0.30

Ervand Kogbetliantz, USA

770. K. F. Teodorchik, "Thomson's self-excited vibrating systems of third and fourth order" (in Russian), *J. tech. Phys.* (*Zh. tekh. Fiz.*), Oct. 1947, vol. 17, pp. 1165–1170.

This paper is a study of the application of the Routh-Hurwitz stability criteria to quasi-linear cases of the third and fourth order, on the premise that if a self-excited vibrating system is

close to being a conservative linear system, its vibrations will be almost harmonic.

After considering a second-order case, the author extends the use of the Routh-Hurwitz criteria to an inertially nonlinear equation of the third order

$$\psi_o(x^2)\,\dddot{x} + \psi_1(x^2)\,\ddot{x} + \psi_2(x^2)\,\dot{x} + \psi_3(x^2)\,x = 0 \quad [1]$$

(where all the $\psi(x^2)$'s are polynomials, dependent on the mean-squared value of the independent variable) and forms the function

$$\Psi(x^2) = \psi_1(x^2)\,\psi_2(x^2) - \psi_o(x^2)\,\psi_3(x^2) \quad [2]$$

The conditions for self-excitation of the system will be $\psi_i(0) > 0$ and $\Psi(0) < 0$. The possible steady-state amplitudes and the conditions for them, as well as the frequencies, are then found. In the absence of self-starting the system will be potentially self-vibrating and may have pure harmonic steady-state regimes, stable under the same conditions as for self-excitation.

The author next considers an inertialessly nonlinear system of the type

$$\psi_0(x)\,\dddot{x} + \psi_1(x)\,\ddot{x} + \psi_2(x)\,\dot{x} + \psi_3(x)\,x = 0 \quad [3]$$

and, again using the Routh-Hurwitz criteria, forms the function

$$\Psi(x) = \psi_1(x)\,\psi_2(x) - \psi_o(x)\,\psi_3(x) \quad [4]$$

and gives the conditions under which the system will have almost harmonic vibrations. He states that he may obtain these conditions by the method of Van der Pol or by passing from Equation [3] to Equation [1]. An example employing a vacuum-tube resistance-capacitance oscillator is considered, and the conditions for self-excitation are found.

By use of the Routh-Hurwitz criteria the conditions for the existence of a pure harmonic solution and stability of all other partial solutions for a nondegenerative linear system of the fourth order are found. The author then states without elaboration that the behavior of nonlinear self-excited vibrating and potentially self-excited vibrating systems of the fourth order will be determined by the proximity with which these systems approach the nondegenerative linear system of the fourth order.

John M. Kopper, USA

771. N. Minorsky, "Self-excited mechanical oscillations," *J. appl. Phys.*, Apr. 1948, vol. 19, pp. 332–338.

In this paper, the author considers oscillatory phenomena which can be represented by the quasi-linear difference-differential equation

$$\ddot{\theta} + p\dot{\theta} + q(\lambda)\dot{\theta}_h + \omega_o^2\theta + \mu f(\dot{\theta}, \dot{\theta}_h) = 0$$

where p and ω_o^2 are constants, q is a function of a parameter λ of the system, f is a nonlinear function, and μ is a parameter of the system having a small value, corresponding to small nonlinearities in the system. $\dot{\theta}_h = \dot{\theta}(t-h)$ is a retarded velocity upon which the damping depends, t being the time and h a constant time lag.

The existence of harmonic oscillations for discreet values of q is shown by first considering the linearized problem ($\mu = 0$). To determine the stability and the stationary amplitudes of these oscillations, the author then investigates the nonlinear equation for values of the parameters in the neighborhood of those that yield the lowest harmonic mode for the linear problem.

Assuming the solution to have the form $\theta = a \sin(\omega t + \beta)$ and $\dot{\theta} = a\omega \cos(\omega t + \beta)$ where a and β are unknown functions of time, the average rate of change of the amplitude da/dt, and of the phase $d\beta/dt$ are computed to a first approximation. The stationary amplitude is then found by setting $da/dt = 0$, and is shown to be determined uniquely by the parameters of the system and not by the initial conditions.

The analysis is applied to the problem of the antirolling stabilization of ships, where it is assumed that $f(\dot{\theta}, \dot{\theta}_h) = -q_3\dot{\theta}_h^3$. For this case, the self-excitation, the stationary amplitude, and the stability of the stationary amplitude are investigated. The author states that the theory is in agreement with experiments.

Albert I. Bellin, USA

772. Walter T. White, "An integral-equation approach to problems of vibrating beams, Parts I, II," *J. Franklin Inst.*, vol. 245, Jan. 1948, pp. 25–36; Feb. 1948, pp. 117–133.

The author contends that problems in vibrations of beams are handled best by means of integral (rather than differential) equations. The "kernels" of these integral equations are the "Green's functions" or "influence functions," that is, the deflections at one point caused by a unit load at another point. Green's functions are constructed by the author explicitly for a uniform cantilever, and as definite integrals for nonuniform beams and for naturally twisted beams. The integral equations are obtained by equating the deflections y due to the inertia loads $-m\omega^2 y$. To solve the integral equations a method of successive substitutions is used. The method is illustrated on a twisted turbine blade. A cinema integraph was used in evaluating the kernels.

This reviewer, while admitting the power of the integral-equation method when properly applied, believes that where special integrating instruments are unavailable the direct integration of the differential equations by the method of M. A. Prohl or N. O. Myklestad has much to recommend it.

H. Poritsky, USA

Wave Motion, Impact, Seismology

(See also Revs. 762, 827, 836, 856, 874, 911, 916)

773. C.-G. Rossby, "Notes on the distribution of energy and frequency in surface waves," *J. mar. Res.*, June 1947, vol. 6, pp. 93–103.

In this note, the behavior of irrotational water surface waves having small amplitudes is used to demonstrate the fundamental characteristics of energy distribution in dispersive media. The analysis in the first part of the investigation indicates that there is no transfer of the wave energy from one portion of the frequency spectrum to the other.

The second part of the paper is concerned with the study of a train of progressive surface waves in which the group velocity of the forward portion is less than that of the rear portion. It is demonstrated that, in such a case, a sharp progressive wave front may be formed in which the wave period increases with the time. The mathematical results are interpreted graphically.

Louis A. Pipes, USA

774. M. Levenson and B. Sussholz, "The response of a system with a single degree of freedom to a blast load," *David Taylor Model Basin Rep.*, no. 572, Dec. 1947, pp. 1–24.

An analysis of systems possessing a single degree of freedom is conducted to determine the response to single blast loads and to two identical blast loads acting at different times.

Plots of the dynamic load factor of the system with damping indicate lower loads than with no damping. Similar plots of dynamic load factor for systems under successive impulses (such an analysis was motivated by the authors' experience in analyzing gun-blast records) lead to higher design loads. A plot

which is useful for design gives the relation between the peak blast pressure and the positive impulse necessary for a given maximum displacement.

The natural frequencies of plates and beams with various edge supports are also tabulated. George Gerard, USA

775. **E. Z. Stowell, J. C. Houbolt, and S. B. Batdorf, "An evaluation of some approximate methods of computing landing stresses in aircraft,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1584, May 1948, pp. 1–32.

The purpose of this paper is to obtain some idea as to the possible errors involved in making certain simplifying assumptions when the landing loads on an airplane are calculated. The airplane was therefore simplified to such an extreme that an exact solution became possible.

For this simplified system, in which the airplane was considered as a uniform beam with a concentrated mass at its middle to which two linear springs were attached, the conclusions were drawn that the airplane may be considered as a rigid body for the purpose of finding the forces or accelerations at the landing-gear attachments, but that the flexibility of the structure must be considered for determining the actual bending moments and shears. N. O. Myklestad, USA

Acoustics

(See also Rev. 900)

776. **C. P. Brittain, C. R. Maguire, R. A. Scott, and A. J. King, "Attenuation of sound in lined air ducts,"** *Engineering*, Jan. 30, 1948, vol. 165, pp. 97–98; Feb. 13, 1948, pp. 145–147.

Experimental results are presented on the attenuation of sound in ducts due to: (1) The physical dimensions of the duct and lining; (2) the effect of propagation around 30, 60, and 90-deg corners; and (3) the presence of resonant absorbers, either lined or unlined, along the walls of the duct. The data presented indicate that the large attenuation with low-density material predicted by the theory of Morse [*J. acoust. Soc. Amer.*, 1939, vol. 11, p. 205] in the range of 500 to 1000 cycles is not attained in practice due to transmission of sound in the lining itself.

Corners are found to introduce additional attentuation, which varies irregularly from 0 to 10 db below 2000 cycles, and increases to 35 db at 5000 cycles for certain conditions. Resonant absorbers along the walls of a duct are found to give attenuations of from 4 to 9 db per ft, but are limited in their effectiveness to one or two frequencies. J. R. Frederick, USA

777. **Paul Wirz, "Properties of simple and multiple cylindrical acoustic resonators (Eigenschaften von ein- und mehrfachen zylindrischen akustischen Resonatoren),"** *Helv. phys. Acta.*, 1947, vol. 20, no. 1, pp. 3–26.

To investigate the characteristic properties of acoustic resonators and to explain the fine structure of their resonance curve (r.c.) the author measured the resonance curves of many resonators of various dimensions and having different mouth openings. The results show that the resonance frequency (r.f.) decreases with contraction of the resonator mouth, whereas the absorption coefficient (a.c.) increases. On the other hand, for third harmonics the a.c. decreases with the contraction of the mouth and the resonance peaks are shifted toward smaller wave lengths.

The author concludes that the fine structure of the resonance curve is due to the superposition of the r.c. of the resonator and the space r.c. resulting from the reflected wave from the boundaries. Measured values of the a.c. with various mouth openings are compared with calculated values. The results agree to within one per cent of the observed resonance wave lengths of the resonators. The measurements are given in a table for 24 resonators of different dimensions and neck width.

The author also made measurements of the pressure distribution inside and in front of the resonator mouth. Nicholas Chako, USA

778. **W. Furrer, "Testing of insulating materials against sound due to walking (Die Untersuchung von Trittschall-Dammstoffen),"** *Schweiz. Bauztg.*, Dec. 27, 1947, vol. 65, pp. 711–714.

The effectiveness of a damping material in preventing the transmission through building structures of impact sounds, such as those due to walking, depends on its elasticity and frictional damping.

The author has devised an apparatus which measures these quantities at various frequencies for materials under combined static and dynamic loading. The elasticity and frictional damping are obtained from a resonance curve. A unique feature of the apparatus is that the static loading and resonance frequency can be independently varied. Results are given for pads of coconut fibers, sponge rubber, textile fibers, and glass fibers under static loads up to 700 kg per sq meter, and for frequencies up to 100 cps. Richard K. Cook, USA

779. **F. G. Tyzzer and H. C. Hardy, "The properties of felt in the reduction of noise and vibration,"** *J. acoust. Soc. Amer.*, Sept. 1947, vol. 19, pp. 872–878.

This paper presents the results of measurements of the dynamic stiffness and damping of various grades of felt by means of resonance curves. The use of felt in a vibration-isolation mounting depends on the values of these quantities, and it is particularly emphasized that it is the dynamic stiffness and not the more commonly measured static stiffness which is relevant to this problem. The measurements include a study of the dependence of the above-mentioned parameters on time, pad dimensions, amplitude of motion, and pressure.

Among the conclusions reached is the fact that a felt mounting is not particularly suitable for the isolation of very low frequency vibration, namely below 40 cps. Adequate curves are presented showing the measured functional relations over rather wide ranges of the independent variables. R. B. Lindsay, USA

780. **Vilhelm L. Jordan, "The application of Helmholtz resonators to sound-absorbing structures,"** *J. acoust. Soc. Amer.*, Nov. 1947, vol. 19, pp. 972–981.

Resonance absorption of Helmholtz resonators combined with ordinary absorption of materials are studied. Since perforated structures or walls provide selective absorption, the author gives a simple theory of a single resonator for plane waves. The most important case is absorption due to viscosity, although radiation resistance is not negligible when one employs diffusion materials.

The predicted absorption, due to the resistance of the neck of the resonator, is much smaller than the experimental value. Partial correction is obtained by applying the results of Sivian [*J. acoust. Soc. Amer.*, 1935, vol. 7, p. 94] and introducing an external resistance of the neck of the resonator in addition to its internal resistance The large residual discrepancy is explained in part by the inaccuracy of the tube method, as the resistance increases for high particle velocity, and thus the absorption depends on the pressure. In addition, turbulence would further increase the absorption when high pressures are involved. If the slits of

the resonator are narrow, but longer than the wave length of sound, radiation resistance becomes effective and, according to Pederson, the absorption of such resonators is greater than for the previous case. This type of resonator shows a selective absorption as all Helmholtz resonators do.

Measurements with different size resonators, alone and combined with an absorbing layer, show that the latter combination gives a much better soundproof hall than the absorption material alone. It is found that the absorption is 90 per cent at 50 cycles, resulting in an improvement for low frequencies. Finally, from observations in Copenhagen, the author concludes that by combining resonators with sound absorbers one obtains better reverberation characteristics, especially for low frequencies. Graphs and sketches of several types of resonators are presented.

Nicholas Chako, USA

781. P. Rothwell, "Calculation of sound rays in the atmosphere," *J. acoust. Soc. Amer.*, Jan. 1947, vol. 19, pp. 205–221.

Fujiwhara's method of tracing sound rays in a moving atmosphere is refined. It is assumed that the velocity of sound (relative to the ground) at any point is dependent only on the air temperature and wind velocity. These in turn are assumed to be functions of the altitude only, and in addition the wind bearing is supposed to be constant. An arbitrary distribution of sound velocity and wind speed is handled by dividing the atmosphere into horizontal layers, within each of which the sound velocity and wind speed vary linearly with altitude.

Tables for facilitating the computation of ray paths are given, and some simple examples are treated in detail.

Richard K. Cook, USA

782. J. W. Miles, "The equivalent circuit for a bifurcated cylindrical tube," *J. acoust. Soc. Amer.*, July 1947, vol. 19, pp. 579–584.

This is a purely mathematical paper which applies the equivalent electric-circuit method to the solution of the problem of the propagation of sound waves through a cylindrical tube in which another finite cylindrical tube of smaller cross section has been inserted. An acquaintance with the author's previous papers ["The analysis of plane discontinuities in cylindrical tubes," *J. acoust. Soc. Amer.*, 1946, vol. 17, pp. 259–272; pp. 272–285] is essential to a complete understanding of the present paper.

Application is made to the special case of concentric circular tubes, and susceptance curves are calculated as functions of the ratio of tube radii, for values of the ratio of the larger tube radius to the wave length ranging from 0 to 0.5.

R. B. Lindsay, USA

Elasticity Theory

(See also Revs. 763, 766, 796, 823)

783. N. M. Newmark, "Influence charts for computation of vertical displacements in elastic foundations," *Univ. Ill. engng. Exp. Sta. Bull.*, no. 367, Mar. 23, 1947, vol. 44, pp. 7–11.

This bulletin presents three charts which are designed to determine the vertical displacement in an elastic, homogeneous, isotropic, semi-infinite solid body loaded vertically by a distributed load on its horizontal surface. Chart 1 indicates the vertical displacement on the surface of the elastic body. Chart 2 indicates the vertical displacement at a depth z beneath the surface, assuming that the elastic body has a Poisson's ratio of 0.5. Chart 3 indicates the vertical displacement at a depth z beneath the surface, with correction values for Poisson's ratios other than 0.5.

The construction and use of these influence charts, including an illustrative example, are discussed. R. K. Bernhard, USA

784. Chih-Bing Ling, "The stresses in a plate containing an overlapped circular hole," *J. appl. Phys.*, Apr. 1948, vol. 19, pp. 405–411.

Supplementing the previous paper by the same author, "On the stresses in a plate containing two circular holes" (see Rev. 411, Applied Mechanics Reviews, Mar. 1948), a solution is given for the case where the two holes overlap. The stress function concerned is constructed by means of an even integral solution of the biharmonic equation expressed in bipolar co-ordinates. The parameters involved in the solution are determined from the given boundary conditions with the aid of Fourier transforms.

Expressions for the stress along the rim of the hole are derived for three stress systems, all-round tension, longitudinal tension, and transverse tension. Some numerical results are presented.

It is shown that the expressions for the maximum stress in the limiting case corresponding to two complete circular holes tangential to each other are the same as obtained in the previous paper. R. G. Boiten, Holland

785. D. J. Scherman, "Solution of problems in theory of elasticity for doubly connected regions (Sur une méthode de résoudre certains problèmes de la théorie de l'élasticité pour les domaines doublement connexes)," *C. R. Acad. Sci. URSS*, Mar. 20, 1947, vol. 55, pp. 697–700.

In this paper the author considers the plane problem in the theory of elasticity for a doubly connected region S, whose boundary L consists of two simple closed curves L_1 and L_2. As is known, this problem is reducible to the determination of two functions $\varphi(z)$ and $\psi(z)$ which are analytic inside S, and which, on the boundary L, satisfy the relation

$$\varphi(t) + t\,\overline{\varphi'(t)} - \overline{\psi(t)} = f(t)$$

where t is the parameter to which points of the boundary are referred, and $f(t)$ is a given function.

In the present paper this problem, by the introduction of an auxiliary function which is defined on L_1 (or L_2), is reduced to an equivalent problem for a simply connected domain. The auxiliary function here referred to is characterized by a Fredholm equation.

R. G. Boiten, Holland

786. J. L. Synge, "The method of the hypercircle in elasticity when body forces are present," *Quart. appl. Math.*, Apr. 1948, vol. 6, pp. 15–19.

This paper is a generalization of an earlier paper [W. Prager and J. L. Synge, "Approximations in elasticity based on the concept of function space," *Quart. appl. Math.*, Oct. 1947, vol. 5, pp. 241–269] dealing with the method of the hypercircle in linear function spaces for the approximation of the boundary-value problem in elasticity with body forces absent. It is shown that the method of the hypercircle can be applied in general to the problem of elasticity when body forces are present. The formulation of the method is similar to that given in the above-mentioned paper.

The natural state of stress in an elastic body in equilibrium, in the presence of body forces, is determined by the six stress components as functions of position. They satisfy: (*a*) Three equations of equilibrium; (*b*) three equations of compatibility throughout the body; and (*c*) various boundary conditions on different portions of boundary surfaces. These six functions determine a vector **S** in a metric manifold characterized by the positive definite

form representing the strain-energy function, in which the generalized Hooke's law for an anisotropic medium is used. The length S of **S** is defined in such a manner that $^1/_2S^2$ is equal to the total strain energy in the elastic body.

The author considers approximate states **S** in which one or more of the conditions (a), (b), (c) are relaxed. It is shown that the natural state **S** lies on a hypercircle determined by two classes of such approximate states. Hence any approximate state inside or on the hypercircle thus determined can be used as an approximation of the natural state, whose strain energy is bounded below and above by this hypercircle. The accuracy of the approximation can be improved by various choices of approximate states, so that the radius of the hypercircle thus determined is made smaller and smaller until the required accuracy is obtained.

Wei Zang Chien, China

787. Carlo Cattaneo, "Theory of the elastic contact in second approximation (Teoria del contatto elastico in seconda approssimazione)," *R. C. Mat. Appl.*, Dec. 1947, vol. 6, pp. 504–512.

Two surfaces of revolution (to which this paper is limited) are in contact at O. If a plane is drawn tangent to the surfaces at O, the distance from a point in the plane a distance ρ from O to one of the surfaces is represented by expressions of the type $a_1\rho^2 + b_1\rho^4$. Following a derivation similar to the one used by Herz (who considered only the term $a_1\rho^2$), a finite solution for the pressure and contact stresses is found.

The pressure distribution is found to have the following form

$$m\left(1 - \frac{\rho^2}{a^2}\right)^{1/2} + n\left(1 - \frac{\rho^2}{a^2}\right)^{3/2}$$

where a is the radius of the contact area and m and n are two parameters determined by the shape of the surfaces, the load, and the physical constants of the material. No bibliography is included.

John E. Maulbetsch, USA

788. M. A. Sadowsky and E. Sternberg, "Stress concentration around an ellipsoidal cavity in an infinite body under arbitrary plane stress perpendicular to the axis of revolution of cavity," *J. appl. Mech.*, Sept. 1947, vol. 14, pp. 191–201.

As can be expected the problem indicated in the title is solved by the introduction of an orthogonal curvilinear co-ordinate system

$$x = x(\alpha, \beta, \gamma)$$
$$y = y(\alpha, \beta, \gamma)$$
$$z = z(\alpha, \beta, \gamma)$$

chosen in such a way that the surface of the cavity is represented by $\alpha = \alpha_o$.

If the state of plane stress to which the infinite body is subjected is defined by σ_x, σ_y, σ_z and τ_{xy}, τ_{zx}, τ_{zy}, two different basic problems are to be considered, defined by I

$$\sigma_x = \sigma_y = 1,\ \sigma_z = \tau_{zx} = \tau_{zy} = 0 \text{ at infinity}$$
$$\sigma\alpha = \tau\alpha\beta = \tau\alpha\gamma = 0 \text{ at } \alpha = \alpha_o$$

and by II

$$\sigma_x = -\sigma_y = 1,\ \sigma_z = \tau_{zx} = \tau_{zy} = 0 \text{ at infinity}$$
$$\sigma\alpha = \tau\alpha\beta = \tau\alpha\gamma = 0 \text{ at } \alpha = \alpha_o$$

Every other problem involving plane stresses at infinity can be looked upon as a linear superposition of the two foregoing ones.

In each of the cases I and II uniform stress field is first extended throughout the whole space (so that the conditions at the cavity boundary are violated), and subsequently solutions are sought which remove the residual stresses at $\alpha = \alpha_o$ and leave the stresses at infinity undisturbed. An exact closed solution is found. The technically important features of the ensuing stress concentration are fully discussed.

C. B. Biezeno, Holland

789. H. J. Greenberg and Rohn Truell, "On a problem in plane strain," *Quart. appl. Math.*, Apr. 1948, vol. 6, pp. 53–62.

Applying a method recently developed by W. Prager and J. L. Synge [*Quart. appl. Math.*, Oct. 1947, vol. 5, p. 241], the authors study the case of the uniform lateral elastic compression of a long rectangular prism, long enough to be assumed in a state of plane strain. The cross section of the prism is square, Poisson's ratio is 1/3, and the friction is assumed to be large enough to prevent any strains in the plane of the loaded boundaries. The authors found that the relation between the applied load per unit length of bar F, the total contraction $2a$ in the direction of the load, and the modulus of elasticity E of the material is $E = 0.4159\ F/a$. In terms of average stress σ and strain ϵ in the direction of the load this expression is $E = 0.8318\ \sigma/\epsilon$ which compares with the value $E = 0.8889\ \sigma/\epsilon$ for pure compression in plane strain with no friction on the loaded surfaces.

This study does not solve the important material-testing problem of the correction to be introduced when cubes are tested under compression to determine E. However, this study, with previous work not mentioned in the paper, such as that of Knein [*Abh. Aero. Inst. T. H. Aachen*, no. 7, 1927], is an important contribution to the solution of that problem.

A. J. Durelli, USA

Experimental Stress Analysis

790. J. H. Lamble, "Experimental method of stress analysis," *Engineering*, May 7, 1948, vol. 165, p. 436.

To study stress distribution in plates under hydrostatic loading, a brittle coating was applied to their surface with ordinary plumber's resin applied by rubbing the hot surface with a large lump. The pictures published are clear, with well-defined cracks, but no quantitative evaluation of errors is given. The author claims that this coating is not sensitive to reasonable changes in temperature and humidity, and that its use with solvents is not advisable. These tests, like many others published in Europe, contradict American experience.

A. J. Durelli, USA

791. C. Mylonas, "The optical system of polariscopes as used in photoelasticity," *J. sci. Instrum.*, Mar. 1948, vol. 25, pp. 77–81.

In photoelastic polariscopes it is often assumed that the field of light between condenser and collector lenses is parallel. This permits placing the specimen anywhere between these two lenses. Actually this can be true only for point-source light. The author analyzes the optical system of the polariscope and finds that the specimen should be as close as possible to the collector lens. He also studies the thickness effect and the influence of size of specimen on the sharpness of the isochromatics. He shows that the size and focal length of the lenses and the size and brightness of the source should be chosen according to the size and thickness of the specimens.

A. J. Durelli, USA

792. N. R. Goncharov, "The brittle lacquer method of determining stresses," *Metallurgia*, Apr. 1948, vol. 37, pp. 290–292 (abstracted from book "Stress determination in machine parts by tensometers and lacquers," Leningrad-Moscow, 1946).

Chapter 4 of Goncharov's book (abstracted by G. S. Smith) is the first publication on this subject from Russia available in

English. It summarizes work of the author and of other Russian researchers: Fisher, Shikhobalov, Prokofev, Makarov, and Andreevsky. The main contribution is in the results described of tests with a large number of coatings of different compositions. These results, however, are less satisfactory than those obtained in the United States with "Stresscoat." The scattering of the values is larger, and the strain sensitivity is smaller. Andreevsky found an influence of barometric pressure on the behavior of the coating, a phenomenon not noticed before. He also found that the coating is better spread with a brush, which is contrary to general experience here. A. J. Durelli, USA

793. J. A. Bennett and H. C. Vacher, "Calibration of x-ray measurement of strain," *J. Res. nat. Bur. Stands.*, Apr. 1948, vol. 40, pp. 285–293.

This paper describes a procedure for improving the precision of strain determination by the two-exposure x-ray method, for uniaxial stress. Patterns were made at incidence angles of 45 and 90 deg; 12 measurements of ring radius were made on each pattern. The tangent of twice the diffraction angle θ is plotted against the ratio of strain at an angle (corresponding to a point of measurement) to strain parallel to the surface of the specimen in the direction of stressing. This plot is fitted with a straight line and extrapolated to the strain at zero angle. This maximum strain varies linearly with tan 2θ throughout the elastic range, to a close approximation.

A variation of stress of 1000 psi in a flat steel specimen is detectable by this method. A. R. Bobrowsky, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 812, 820, 821, 824)

794. B. Fritz, "Simplified method for the calculation of minimum height of continuous steel girders with variable moments of inertia (Vereinfachtes Verfahren zur Ermittlung der Mindestträgerhöhe durchlaufender Stahlträger mit beliebig veränderlichen Trägheitsmomenten)," *Bauplan. Bautech.*, Feb. 1948, vol. 2, pp. 48–50.

The paper describes an approximate design procedure for proportioning variable section continuous girders with particular emphasis on keeping deflections within specified limits. The method is based on formulas developed by the author but their derivation is not given. Dana Young, USA

795. J. G. McGinley, "Static structural tests of eleven lightweight anchors," *David Taylor Model Basin Rep.*, no. R-349, Jan. 1948, pp. 1–33.

Each anchor was subjected to tensile loads applied at the shackle eye of the shank and at the extreme tips of the flukes, simulating the most severe service condition. Strains at all critical sections were measured by SR-4 wire gages. The load was raised until the anchor fractured or until it sprang free from the gripping clamp. Relative merits of the types of anchors tested are discussed, with comments for redesign.

Evan A. Davis, USA

796. E. A. F. Huber, "Design of rectangular beams of reinforced concrete under bending stress with consideration of the tension in the concrete (De dimensionering van rechthoekige, op buiging belaste balken van gewapend beton met inachtneming van de trekspanningen in het beton)," *Ingenieur 's-Gravenhage*, Mar. 5, 1948, vol. 60, pp. Bt. 11–18.

Based on the assumption that the cross sections of the beam remain plain and taking into account the fact that the moduli of elasticity for tension and for compression depend on the compression strength and on the resulting extreme fibre stresses (according to Scheyer's investigations), a system of design formulas is derived for beams of reinforced concrete, loaded by pure bending as well as by bending and axial forces.

Some tables for aid in design and some examples illustrating the procedure are included. W. L. Esmeijer, Holland

Plates, Disks, Shells, Membranes

(See also Revs. 804, 805, 806, 807, 808, 809, 810, 811, 813, 814, 815, 816, 823, 835)

797. J. H. Lamble and Li Shing, "A survey of published work on the deflection of and stress in flat plates subject to hydrostatic loading," *Trans. Instn. nav. Archit. Lond.*, Apr. 1947, vol. 89, pp. 128–147.

This paper presents a useful survey of literature on thin plates, whose deflection is small compared with the thickness, under hydrostatical load. The subject is divided into theoretical and experimental contributions. Circular and rectangular plates are considered, with free or clamped support.

Wherever possible the authors give in graphical form comparison of the several theoretical solutions and plot the available experimental data. This comparison is sometimes quite discouraging. After many years of research the authors found no results of experiments on freely supported rectangular plates, and the difference between theory and some experiments in the case of the clamped plate is of the order of 80 per cent. More than 60 bibliographic references are given at the end of the paper.

A. J. Durelli, USA

798. V. V. Sokolovski, "Plastic stressed state of rotating disks" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), Jan.-Feb. 1948, vol. 12, pp. 87–94.

Stress distributions in rotating disks of variable thickness are examined for angular velocities such that the disk is entirely in the plastic range. It is assumed that for a certain critical velocity the disk becomes entirely plastic and that strain hardening appears only at higher velocities (which would presumably only be true for materials like mild steel). The plastic stress state (without strain hardening) is defined by the conventional "energy of distortion" theory. Under these assumptions, the state of stress corresponding to the critical velocity, together with this velocity itself, are theoretically determined, and a numerical procedure for finding them is given, with an example. A simplified method, in which it is assumed that the tangential stress is constant, is also indicated. This leads to closed analytical solutions, the latter being good approximations to the more "exact" method.

For the case of a disk beyond the critical velocity when strain hardening sets in, the author gives only a system of two differential equations, with boundary conditions and general indications for obtaining the solution. Leon Beskin, USA

799. C. Libove and S. B. Batdorf, "A general small-deflection theory for flat sandwich plates," *Nat. adv. Comm. Aero. tech. Note*, no. 1526, Apr. 1948, pp. 1–53.

The authors develop fundamental equations for the bending of sandwich plates consisting of a low-density core bonded between

thin sheets of high-stiffness material. It is assumed that the face sheets and the core may have orthotropic flexural and extensional properties and that the shear deflection of the plate may be appreciable on account of its low-stiffness core. The plate is characterized by seven physical constants—two flexural and two shear stiffnesses, a twisting stiffness, and two Poisson ratios—of which six are independent, while the two Poisson ratios must be in the same proportion as the corresponding flexural rigidities.

For a plate of this type three equilibrium and three compatibility equations are derived, and it is shown that by elimination a single differential equation of the sixth order may be established for any of the six unknowns. On account of shear deflection, the boundary conditions for the plate require the specification of one more variable than in ordinary plate theory.

The authors also derive expressions for the strain energy of deformation in such plates and for the potential energy of external forces, thus providing a basis for investigation of their elastic stability by the energy method. However, no attempt is made to solve any specific problems. M. Hetényi, USA

800. **Karl Federhofer, "The fundamental equations of elastic plates with nonconstant thickness and large deflection (Die Grundgleichungen für elastische Platten veränderlicher Dicke und grosser Ausbiegung),"** *Z. angew. Math. Mech.*, Apr. 1947, vol. 25–27, pp. 17–21.

In this paper the author discusses elastic plates with nonconstant thickness. Two simultaneous partial differential equations for the deflection and the membrane stress function are developed and presented in both Cartesian co-ordinates and in polar co-ordinates.

The author points out a remarkable formal resemblance between the left sides of the two differential equations.

The symmetrically loaded circular plate is particularly discussed. C. J. Bernhardt, Norway

801. **Chi-Teh Wang, "Bending of rectangular plates with large deflections,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1462, Apr. 1948, pp. 1–34.

This is a continuation of previous work of the author [*Nat. adv. Comm. Aero. tech. Note*, no. 1425, 1948] in which the general theory and the case of a square plate of uniform thickness under uniform normal pressure are discussed. The present paper investigates the large deflections under uniform pressure of rectangular plates with aspect ratio 1.5 and 2.

These papers differ from previous investigations of other authors in two respects: (1) The boundary conditions are formulated for a single panel with riveted edges, the normal displacement and the bending moment being assumed to be zero at the edges, and the strain along the edges and the displacement in the plane of the plate normal to the edges being also assumed to be zero. Thus the boundary conditions are a mixture of those usually given for simply supported and for clamped edges. (2) The actual calculation is done by transforming all the equations into difference equations and solving these equations by iteration.

Numerical data are given for values of pa^4/Eh^4 up to 250, where p is the normal pressure, a the length of the shorter side of the plate, E Young's modulus, and h the plate thickness. H. S. Tsien, USA

802. **H. D. Conway "The bending of symmetrically loaded circular plates of variable thickness,"** *J. appl. Mech.*, Mar. 1948, vol. 15, pp. 1–6.

The author solves the differential equation for bending of thin circular plates of variable thickness under symmetrical loading, when the thickness is proportional to the distance from the plate center. Coefficients for determining the maximum stress and deflection are tabulated for six loading cases and six values of the ratio of the outer edge radius to the inner (clamped) edge radius. Comparison is made with the corresponding results for a plate of constant thickness. Some numerical errors occur in the table. P. C. Dunne, England

803. **Sigge Eggwertz, "Theory of elasticity for thin circular cylindrical shells,"** *Trans. roy. Inst. Technol. Stockh.*, 1947, no. 9, pp. 1–25.

The paper summarizes the European development of the engineering design of cylindrical shells. Various very approximate and more exact methods of analyzing membrane and bending stresses are described. The risk of buckling is mentioned only in passing.

A detailed discussion is given of the particular solutions for the displacement components

$$u = U(\varphi)\cos\lambda x$$
$$v = V(\varphi)\sin\lambda x$$
$$w = W(\varphi)\sin\lambda x$$

where u is axial (x direction), v is circumferential (φ direction), and w is radial. All the forces and deformations can be written as functions of W.

No reference is made to any American literature, but a bibliography of 31 references includes interesting recent publications of Scandinavian authors. D. C. Drucker, USA

Buckling Problems

(See also Revs. 799, 826)

804. **A. A. Ilyushin, "Stability of plates and shells beyond the proportional limit,"** *Nat. adv. Comm. Aero. tech. Memo.*, no. 1116, Oct. 1947, pp. 1–44 [transl. from *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, 1944, vol. 8, p. 337].

The author assumes a plastic material with strain hardening, plastic deformations being governed by the shearing energy, a one-to-one correspondence between stress and strain being assumed. After establishing the relations between the infinitesimal stresses and strains produced by buckling, the differential equation governing the stability of plates is derived. Some simple cases of stability of plates and shells are then investigated.

The present reviewer has followed the same line of reasoning in several previous papers [*Proc. kon. Ned. Akad. Wet.*, 1938, vol. 41, p. 468 and p. 731; *Publ. int. Ass. Bridge Struct. Engng.*, 1940–41, vol. 6, p. 45] and came to many of the conclusions reached by the author, such as the anisotropic behavior of the material during buckling and the fact that the flexural rigidity of a plate at the yield stress is not zero. The author considers the boundary surface between the elastic and plastic regions in the buckled plate in more general manner, but in deriving the differential equations of the plate he assumes that the resultant normal and shearing forces in buckling do not vary, which means that this boundary surface is parallel to the middle surface. The reviewer found this not to be the case even for the simple case of a wide plate with two free edges.

The author starts from the same assumption as is used in the theories of Engesser and von Kármán for bars, that no bending occurs before the critical load is reached, which Shanley has shown results in too high a critical load [*J. aero. Sci.*, 1947, vol. 14, p. 261]. The author also uses a Poisson's ratio of 0.5 for

elastic as well as plastic deformations. The reviewer's results indicate that this yields flexural rigidities and plastic buckling stresses up to 25 per cent and 10 to 20 per cent too high respectively [*Inst. Baustat. T. H. Zurich*, no. 21].

P. P. Bijlaard, Holland

805. Elbridge Z. Stowell, "A unified theory of plastic buckling of columns and plates," *Nat. adv. Comm. Aero. tech. Note*, no. 1556, Apr. 1948, pp. 1–31.

The author studies the buckling of thin rectangular plates compressed in one direction by a uniform edge loading σ_x. Both elastic buckling and plastic buckling are treated, the latter involving the secant and tangent moduli E_{sec} and E_{tan} obtained from the stress-strain curve. Poisson's ratio is taken as 0.50. In the elastic range the usual differential equation is used: $D\nabla^4 w = -h\sigma_x \partial^2 w/\partial x^2$ with an equivalent formulation by means of energy integrals, from which a characteristic value of the critical elastic buckling stress is found.

In the plastic case the left side of the above equation and the energy integrals are modified under the assumption that the plate is all in the plastic state. Minimization of the latter is accomplished after selecting deflection surfaces satisfying the edge conditions. The ratio η of the critical stresses of the plastic to the elastic case is computed for panels having various edge conditions along the two unloaded edges. The lowest value of η is found to be E_{tan}/E, which occurs when the plate is free at the unloaded edges, so that it buckles as a Euler column and the buckling stresses are bending stresses only. For other edge conditions the value of η is found to vary from E_{tan}/E toward E_{sec}/E, approaching the latter more closely as twisting becomes more important compared to bending.

D. L. Holl, USA

806. F. I. N. Niordson, "Buckling of conical shells subjected to uniform external lateral pressure," *Trans. roy. Inst. Technol. Stockh.*, 1947, no. 10, pp. 1–21.

In this paper a formula is derived by the author for the external fluid pressure p_c which would cause elastic buckling of a thin shell having the form of a circular cone of small taper. For vanishing taper this reduces to von Mises' formula for the cylindrical shell.

Strains and curvatures are obtained in terms of displacements by direct geometrical considerations. Expressions for the strain energy developed and the work done in buckling are then derived. The method for determining p_c is to equate these two as the condition of neutral equilibrium, and to assume forms for the displacement and adjust parameters describing them so as to minimize p_c.

The assumed displacement has one half wave in the axial direction, and n (determined by the minimization) waves circumferentially, all sinusoidal. The justification cited for this assumption is experience with shells having a length-to-radius ratio which is not too large. The energy expressions do not go beyond second-degree terms in the displacement functions and their derivatives.

The application contemplated is to submarine hulls. A reduced modulus is proposed when the proportional limit is exceeded.

J. N. Goodier, USA

807. E. H. Schuette and J. C. McDonald, "Prediction and reduction to minimum properties of plate compressive curves," *J. aero. Sci.*, Jan. 1948, vol. 15, pp. 23–27.

The paper gives an empirical method for predicting the compressive properties of plates by using the similarity that exists between these properties and the stress-strain curve for the material. It is indicated, for example, that the maximum average stress is related to the stress-strain curve in a fairly consistent manner. Experiments show that the method is satisfactory for extruded materials. There are no check data for sheet materials.

J. Hadji-Argyris, England

808. E. H. Schuette, "Buckling of curved sheet in compression and its relation to the secant modulus," *J. aero. Sci.*, Jan. 1948, vol. 15, pp. 18–22.

From the results of a series of compression tests on 87 curved magnesium-alloy panels with length-to-width ratio near unity and radius-to-thickness ratios from 86 to 515, the author suggests the empirical formula for the buckling stress $f_{cr} = 0.42\, E_s(t/R)$. E_s is the secant modulus, t the thickness, and R the radius of the panel.

The statement that the failing stress for nearly all panels was practically identical with the buckling stress is surprising in the case where the recorded buckling stress was low in comparison with the compressive yield stress.

J. Hadji-Argyris, England

809. S. B. Batdorf, Manuel Stein, and Murray Schildcrout, "Critical stress of thin-walled cylinders in torsion," *Nat. adv. Comm. Aero. tech. Note*, no. 1344, June 1947, pp. 1–26.

This is the third of a series of papers [S. B. Batdorf, *Nat. adv. Comm. Aero. tech. Note*, nos. 1341 and 1342, June 1947] dealing with stability calculations for thin cylindrical shells. The authors apply the general method given in the second reference to the particular problem of determining critical stresses in torsion.

Their modified equation defines exactly the same critical stresses as Donnell's equation [L. H. Donnell, *Nat. adv. Comm. Aero. tech. Rep.*, no. 479, 1933] but proves to be more amenable to mathematical treatment by Fourier series techniques.

The critical shear stresses are calculated for simply supported edges and clamped edges by means of Galerkin's method. Although this technique leads to infinite determinants, it is shown that approximations formed by taking only a finite number of terms converge very rapidly in case the number of circumferential waves is greater than two. A separate calculation is carried out for long cylinders which buckle into two circumferential waves.

The critical stresses obtained are in close agreement with those of Donnell except in a transition region (between cylinder proportions for which curvature is and is not important) where there is a maximum difference of 17 per cent. The new solution is presumably correct here since it checks closely with results in this region obtained by D. M. A. Leggett [*Rep. Memo. aero. Res. Comm. Lond.*, no. 1972, 1943], and Donnell's solution was admittedly quite approximate in this region.

G. H. Handelman, USA

810. N. J. Hoff, B. A. Boley, and B. Klein, "Stresses in and general instability of monocoque cylinders with cutouts; III. Calculation of the buckling load of cylinders with symmetric cutout subjected to pure bending," *Nat. adv. Comm. Aero. tech. Note*, no. 1263, May 1947, pp. 1–31.

In this paper an approximate theoretical method is given for calculating the critical bending moments required to cause general instability in cylinders which are reinforced by rings and stringers and have cutouts on the compression side symmetrically placed with regard to the plane of bending. General instability is defined as the simultaneous buckling of rings and stringers together with the sheet attached to them.

The distortion accompanying instability is assumed to be confined to the length of the cylinder occupied by the cutout (this assumption is suggested by experiment) and to extend over a part of the circumference. This part of the circumference is deter-

mined in the calculation; in cylinders with 16 stringers it extends, in the most common case, over 6 spaces between stringers, that is, over 3 spaces on each side of the cutout. Some of the constants in the assumed form for the distortion are found from conditions of continuity at the edge of the buckled part of the cylinder, others from the fact that instability takes place in the mode requiring the least possible bending moment. The bending moment is found by equating the work done by the external forces due to the distortion to the increase in strain energy of the rings, stringers, and attached sheet.

Calculated and experimental critical bending moments for cutout angles of 45, 90, and 135 deg are compared in a diagram. The calculated values are, as is usual, too high (by perhaps 30 per cent), but give approximately the same proportional decrease in bending moment with increasing angle of cutout as the experimental values. W. R. Dean, England

811. **George J. Heimerl, "Methods of constructing charts for adjusting test results for the compressive strength of plates for differences in material properties,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1564, Apr. 1948, pp. 1–14.

Because of the variation in the material properties of different lots of the same material, the results of tests to determine compressive strengths cannot be used directly in design until they are adjusted to the standard material property values on which the design is based.

Methods are here presented for adjusting test results for the critical compressive stress and the average compressive stress at maximum load of flat plates, to standard material property values. The methods take into account the differences between the compression stress-strain curves of the material tested and the standard material. The results are presented in the form of charts giving the multiplying factor to be applied to the test result. Charts for 24S-T and 75S-T extruded aluminum alloys are included. M. V. Barton, USA

812. **A. N. Dinnik and Z. B. Pinskaya, "On the influence of elastic building-in of the ends on the stability of compressed rods"** (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci. (Izv. Akad. Nauk SSSR Ser. tekh. Nauk)*, Dec. 1947, no. 12, pp. 1585–1588.

The authors give the solution for the critical compression on a rod both of whose ends are elastically built in, when the elastic end fixity of the two ends is different. This general case contains as special cases the fundamental cases of columns with ends built in or free to rotate. For various values of elastic end fixity at each end the authors give numerical values and graphical tables for the fixity factor which determines the critical compression.

Z. Bažant, Czechoslovakia

813. **Bernard Budiansky, Pai C. Hu, and R. W. Connor, "Notes on the Lagrangian multiplier method in elastic-stability analysis,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1558, May 1948, pp. 1–46.

A useful mathematical technique for solving problems of stability of elastic plates is illustrated by treatments of several cases of buckling of stiffened and unstiffened rectangular shear-loaded plates with clamped edges. In each case, a virtual deflection pattern is defined by a Fourier series. The plate then acquires virtual strain energy of bending V, and the external forces perform virtual work T.

The solution is based upon the principle that the increment of the quantity $(V - T)$, due to any possible infinitesimal change of the virtual deflection pattern, is zero. A change of the virtual deflection pattern is obtained by giving infinitesimal increments to the Fourier coefficients. However, these increments are not entirely aribitrary, because the Fourier series which defines the virtual deformation does not automatically satisfy the boundary conditions. Consequently, certain numerical series in the Fourier coefficients are equal to zero.

This condition is handled by the Lagrange multiplier method. That is, the numerical series are multiplied by unspecified quantities, and the results are added to $(V - T)$. All derivatives, with respect to the Fourier coefficients, of the modified expression for $(V - T)$ are set equal to zero. Then, after elimination of the Fourier coefficients, certain linear homogeneous algebraic equations in the Lagrange multipliers remain. The buckling stress is determined by the condition that these equations have a nonzero solution.

Plates of infinite length are treated by single infinite Fourier series, and the resulting calculations are comparatively simple. However, plates of finite aspect ratio require the use of double Fourier series, which lead to involved calculations. The authors show how the method may be modified to obtain upper and lower approximations of the solution in cases in which the exact procedure becomes too complicated. H. L. Langhaar, USA

814. **Bernard Budiansky and R. W. Connor, "Buckling stresses of clamped rectangular flat plates in shear,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1559, May 1948, pp. 1–11.

The Lagrange multiplier method, discussed in *Nat. adv. Comm. Aero. tech. Note*, no. 1558, is applied to determine upper and lower bounds for the buckling stress of any flat rectangular isotropic elastic plate which has clamped edges and is uniformly loaded in shear. The bounds are such that the calculated buckling stresses are within $1^1/_4$ per cent of the exact theoretical values.

It is shown that a buckling pattern which is antisymmetrical about the middle point of the plate will occur if the aspect ratio lies within a certain range. H. L. Langhaar, USA

815. **Bernard Budiansky and Paul Seide, "Compressive buckling of simply supported plates with transverse stiffeners,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1557, Apr. 1948, pp. 1–20.

The compressive buckling load of simply supported rectangular plates with identical equally spaced transverse stiffeners is treated by the Rayleigh-Ritz energy method. The effect of torsional rigidity of the stiffeners is taken into account to the extent that the stiffener is considered as a simple torsion member, the possible contribution due to torsion bending being neglected.

Stability criteria are first obtained in series form and then reduced to closed form for the limiting case of infinitely long plates with identical equally spaced stiffeners. Nondimensional charts of the critical loads are given for various aspect ratios of the individual panel. Conrad C. Wan, USA

816. **S. B. Batdorf, "A simplified method of elastic-stability analysis for thin cylindrical shells—I. Donnell's equation; II. Modified equilibrium equation,"** *Nat. adv. Comm. Aero. tech. Note*, June 1947, no. 1341, pp. 1–50; no. 1342, pp. 1–33.

In Part I the author uses a governing partial differential equation of Donnell, which is of the eighth order and omits second and higher degree terms, to develop a method of finding the critical buckling loads for simply supported thin circular cylinders. Solutions are obtained and the results presented graphically for various types of loading—under lateral pressure, axial compression, hydrostatic pressure for a closed cylinder, and torsion. It is

shown that for each case the results may be exhibited as a relation between a stress coefficient k and one geometrical parameter, except when the number of circumferential waves has its smallest value, two.

The method consists of supposing the deflections to be sinusoidal in both the axial and circumferential directions, thence obtaining directly an expression for k in terms of the two wave lengths and minimizing the result; little error is introduced in this process by regarding the circumferential wave length as a continuous variable except when the number of circumferential waves is small. The method is designed to incorporate the required edge conditions for the transverse deflections but is unable to satisfy specified edge conditions of displacement in the direction of the median surface; it is shown however that the automatically implied edge conditions in this surface are of a type which may commonly occur.

In Part II an effort is made to extend the method to deal with problems of cylindrical panels involving shear stresses and also with problems similar to those treated in Part I but with boundary conditions of clamping instead of simple support. The procedure of using double Fourier series expansions for the deflections is found to be more easily facilitated by using a modified form of Donnell's equation. In both parts, results obtained on the basis of this equation are compared with results of other theoretical methods and of experiment. D. N. Allen, England

Joints and Joining Methods

817. T. D. Tuft, "Underwater explosion tests of simple structures fabricated with Everdur brazing," *David Taylor Model Basin Rep.*, no. R-348, Dec. 1947, pp. 1-29.

Tests were made to determine whether Everdur brazing in small-scale models subjected to explosive loading satisfactorily reproduced the types of failures experienced in prototypes with reasonable accuracy, both in type and degree.

It was found that Everdur fillet joints were able to sustain a maximum strain of 10 per cent under explosive loading conducted on simple tray structures made of 0.03-in. black iron sheet, and the geometry of the joint was found to be a deciding factor in determining the type and extent of damage to the sheet. Stiffeners which presented a rounded surface at the junction with the plate provided effective protection against fracture of the plate.

It is of interest that brittle fractures were found to occur frequently in failures of the iron sheet on sizes down to 0.025-in. thickness. T. J. Dolan, USA

818. A. M. Hamilton, "Bolted connections in structures," *Engineering*, May 7, 1948, vol. 165, pp. 433-435.

The author argues that designing for shear in the bolt does not agree with the fact that in many tests there is no mark on either the hole or the bolt shank when the bolt is removed, indicating that even at high loads the members are held by friction. He discusses the use of rivets, bolts with clearance, and force-fit bolts, and recommends clearance bolts for which he finds specification requirements too pessimistic. A. J. Durelli, USA

819. N. Grossman and C. W. MacGregor, "The brittle transition temperatures of various low-carbon steels welded by the same method," *Weld. Res. Suppl.*, May 1948, vol. 13, pp. 267-271.

Results are given of an investigation of the distribution of brittle transition temperatures at points at various distances from the center lines of welds in seven low-carbon steels, including rimmed, semikilled, fully killed, and low-alloy high-tensile steels. All joints were made by the Unionmelt process.

The tests showed the weld metal to be more ductile than the best base-plate material. They indicated the most brittle points to occur 1 in. from the weld center line, about $^1/_2$ in. from the edge of the heat-affected zone.

The following ranges in transition temperatures were observed: weld metal, -95 to -180 F; base plate, -50 to -68 F; most brittle regions, -30 to -55 F. The transition temperature represents here the highest temperature for which the material loses all macroscopic ductile characteristics, as shown by its load-deflection curve at a constant rate of strain. Joseph S. Newell, USA

Structures

(See also Revs. 768, 775, 794, 810, 815)

820. J. Emmen, "A new method for the calculation of suspension bridges (Nieuwe methode voor het berekenen van hangbruggen)," *Ingenieur 's-Gravenhage*, vol. 60, Apr. 23, 1948, pp. B33-40, May 7, 1948, pp. B41-49.

Neglecting the bending rigidity of the stiffening trusses and the elongations of cables and suspenders and assuming the cables to be supported immovably at the towers, the author derives a simple formula for the deflections of the stiffening trusses under live load. The neglected influences are taken into account afterward by iteration.

It is a pity that the author did not omit his assaults on the elasticity theory, as the principle of the deflection theory has been known for 60 years. P. P. Bijlaard, Holland

821. G. Magnel, "The principles of prestressed concrete," *Engng. J. Montreal*, Mar. 1947, vol. 30, pp. 110-112.

This short paper gives a simple and clear explanation of the principles of prestressed concrete. Three most-used methods are described in detail. Photographs of prestressed concrete beams cast at the University of Ghent are shown.

Enrico Volterra, Italy

822. M. Roš, "Interpretation and value of tests and experience with reinforced concrete structures in Switzerland 1924-1947 (Lehre und Nutzen aus den Versuchen und Erfahrungen an ausgeführten Eisenbeton-Bauwerken in der Schweiz)," *Eidgenöss. MatPrüfAnst. Ber.*, no. 99, supplement 5, 1947, pp. 3-47.

The author reviews the developments in reinforced concrete techniques in Switzerland in the last 23 years. He makes a very successful effort to relate theoretical knowledge to experimental observations of laboratory tests, as well as to observations of actual structures. A large bibliography of Swiss papers is included as well as many comments on technological properties of reinforced concrete and on the experimental determination of deformations in structures. A. J. Durelli, USA

823. Leon Beskin, "Warping and shear lag in closed cylindrical shells," *J. aero. Sci.*, Apr. 1948, vol. 15, pp. 221-231.

This analysis of hollow beams with closely spaced rigid frames is based on the assumptions that lateral contraction can be neglected, the material constituting the shell is concentrated along its mean surface, and all plane sections are indeformable in their planes. Two differential equations, one from the condition of compatibility of strain and the other from equilibrium condi-

tions, give the relations between shearing and axial stresses. From these two equations, a method of successive approximation is developed, using conventional beam formulas as the first approximation. In the second approximation, the shear calculated from the equilibrium equation is introduced into the compatibility equation, by which a correction of axial stress is obtained. This new correction value is inserted in the equilibrium equation to obtain the new correction of shear flow, which in turn is used for obtaining a further correction for the axial stresses as the third approximation. This process may be repeated, but the second approximation is generally sufficient for practical problems.

A method for calculating secondary stresses for closed beams under pure torque and pure shear is developed. A detailed sample calculation of a symmetrical rectangular section with one end restrained and with torque applied at the other end is given.

It is shown that the effect of an end restraint or of a change of loading or section properties is to produce a redistribution of shears and of normal stresses. The deviation from elementary theory decreases rapidly, approximately by an exponential law, on both sides of the section where the perturbation occurs. It is noted that this method does not apply for structures stiffened with transverse frames when their spacing is big enough for the secondary stresses to damp to small values between frames. An analysis, following the fundamental method of this paper but introducing the frame spacing in an explicit manner, could be developed for such structures. T. H. Lin, England

824. **J. Lockwood Taylor, "Numerical-graphical method of stressing hollow girders,"** *Aircr. Engng.*, Feb. 1948, vol. 20, p. 34.

This is a very brief presentation of the mathematical relations applicable to analysis of box girders subjected to bending and torsion. It includes relations for symmetrical or unsymmetrical sections with or without taper, with stiffened or unstiffened skin of uniform or varying thickness, on girders carrying distributed or concentrated applied loads.

The paper's value to practicing stress analysts would have been increased a hundredfold by inclusion of an example to illustrate efficient organization of numerical work inherent in applying the method to actual structures. Joseph S. Newell, USA

825. **J. S. Taylor and S. S. Gill, "The analysis of a circular ring with propped floor beam,"** *J. aero. Sci.*, Apr. 1948, vol. 15, pp. 237–242.

The authors consider the solution by energy methods of the redundant case of a circular ring supporting a continuous floor beam, which is typical of problems encountered in aircraft monocoque analysis. A tabular layout to simplify the work is shown. Philip Rosenberg, USA

826. **George Gerard, "Optimum number of webs required for a multicell box under bending,"** *J. aero. Sci.*, Jan. 1948, vol. 15, pp. 53–56.

The author considers a wing or tail surface with a heavy skin in which no buckling can be tolerated. The bending moment is assumed to be taken by the skin alone, stabilized by a number of webs which have to be free of buckling also. No concentrated flanges are supposed to be present.

Proportioning of the cover and the webs on the basis of their critical compression and bending stresses respectively (shear loads are dealt with only qualitatively) leads to an expression for the "solidity" of the structure (ratio of sectional area of structural elements to total area between outer contours). Differentiation with respect to the number of webs yields the condition for minimum weight. The optimum number of webs is found to be a function of the structural thickness ratio only, and is found to be such as to give nearly square cell sections.

H. F. Michielsen, Holland

827. **J. F. McBrearty, "A critical study of aircraft landing gears,"** *J. aero. Sci.*, May 1948, vol. 15, pp. 263–280.

In this comprehensive study the accident statistics collected over the past several years during both civilian and military operations are reviewed and it is shown that of all aircraft structural failures those connected with the landing gear are by far the most numerous. After investigating several possible explanations for this situation, it is concluded that the only valid explanation lies in the unusually severe dynamic loads which can be induced during the landing impact. When standard design procedures are employed these transient loads are accounted for in a purely empirical manner which frequently leads to large design errors.

Experimental data are presented to show that severe landing-gear loads are experienced when fore-and-aft oscillations of the gear are excited by a critical rate of application of wheel spin-up drag forces. The possibility of encountering severe self-excited oscillations under certain circumstances is also discussed. In order to reduce the magnitude of these fore-and-aft oscillations, hydraulic damping of the horizontal wheel motion is advocated; experimental data are given to show the effectiveness of this solution.

The author restricts his attention to the dynamic loads which are induced in the immediate landing-gear structure due to the landing impact. Another facet of this problem which is outside the scope of the paper, but which the reviewer feels should be kept in mind, is the dynamic loading throughout the entire airframe which results from the same cause; this is often responsible for structural failures in areas remote from the gear itself.

Martin Goland, USA

Plastic Flow, Failure; Mechanics of Solid State

(See also Revs. 766, 798, 804, 805, 817, 839, 843, 847, 856)

828. **G. Gurevich, "On the law of deformation of amorphous and polycrystalline bodies"** (in English), *C. R. Acad. Sci. URSS*, Feb. 28, 1947, vol. 55, pp. 493–496.

On the basis of the Maxwell formula

$$\frac{d\sigma}{ds} = k_o - \frac{\sigma}{\tau ds/dt}$$

where σ is the stress and s the strain, and on the assumption that the coefficient τ depends exponentially on σ, a relation is derived between yield stress σ_c and rate of strain v, of the form

$$\log (\sigma_c/v) = A\,\sigma_c - B$$

The author gives experimental confirmation of this relation from a series of tests on a very wide range of materials. He notes that it includes, at the two extremes of range, known simpler relations for the case of rigid materials and the case of liquids of low viscosity. D. N. Allen, England

829. **M. M. Roš, "Fatigue of metals (La fatigue des métaux),"** *Rev. Metall.*, May-June 1947, vol. 44, pp. 125–143.

A review is presented of the character, mechanism, and causes of fatigue failures in ductile materials at normal temperatures

and moderate stress concentrations. The quasi-homogeneity and quasi-isotropy of metals are discussed with the view that materials without faults, dislocations, or internal stresses are nonexistent. Fatigue curves and typical photographs of failures are shown.

From the theory of Mohr, generalized by the Materialprüfungsanstalt in Zürich, a stress parameter

$$\sigma_g = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1}$$

and a strain parameter

$$\epsilon_g = {}^2/_3 \sqrt{\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 - \epsilon_1\epsilon_2 - \epsilon_2\epsilon_3 - \epsilon_3\epsilon_1}$$

are introduced. It is assumed that for a given material σ_g is a function of ϵ_g only. This hypothesis has been satisfactorily verified by static tests of two alloys, Avional and Anticorodal.

The author contends that the risk of fatigue failure under tension, bending, or other kind of stress condition is the same for the same values of σ_g and of the stress gradient $d\sigma_g/dl$.

If the ultimate strengths in tension σ_z and in compression σ_d differ ($\sigma_z < \sigma_d$), the quantity

$$\sigma_v = \sqrt{(\sigma_z/\sigma_d)\sigma_g^2 + \sigma_z(1 - \sigma_z/\sigma_d)(\sigma_1 + \sigma_2 + \sigma_3)}$$

is to be used instead of σ_g. W. Weibull, Sweden

830. E. S. Machlin, "Dislocation theory of the fatigue of metals," *Nat. adv. Comm. Aero. tech. Note*, no. 1489, Jan. 1948, pp. 1–33.

The author develops a theory of fatigue failure for annealed solid solutions based on dislocation theory. This is done by assuming the prior existence of submicroscopic cracks, and postulating certain arbitrary assumptions regarding the nature of the dislocations generated at the ends of the cracks during the stress cycles. The rate of generation of dislocations, obtained from a previous paper by the author and A. S. Nowick, is identified with the net rate of growth per crack, and leads to a semilogarithmic relation between the number of cycles to fracture and the maximum tensile stress of the cycle, for stresses above the so-called endurance limit. The temperature dependence of the process is indicated.

It is assumed throughout that no structural changes take place in the material and that in each cycle the same analysis may be applied. Although the author offers two explanations for the disagreement between theory and experiment indicated by Fig. 7, it seems more likely to the reviewer that, for the temperatures considered, the material had passed through the blue brittle range and as a result strain aging had taken place. The resulting structural changes invalidate the application of the theory.

Edward Saibel, USA

831. Pierre Laurent, "Synthesis of the modern theories on the plasticity of metals (Synthèse des théories modernes sur la plasticité des métaux)," *Rev. univ. Min.*, Mar. 1948, vol. 4, pp. 221–230.

The recent theories of plastic flow in single crystals and polycrystalline structures are reviewed in this paper, and some new ideas are proposed by the author. Included are the well-known theories of Born, Smekal, Taylor, and Becker and Orowan for single crystals. A theory developed by the author, which is somewhat similar to Taylor's dislocation theory, called the theory of "hooking" (accrochage), is discussed. Here faults in the crystal lattice of the type proposed by Smekal are the sources of "hooking," a type of slip mechanism visualized by the author. The "hooking" mechanism develops continuously at a rate governed by stress and temperature, and the "hooks" are displaced through the structure to other flaws where they act to ameliorate the flaw constraint and hence its ability to continue the hooking mechanism at the same rate. This mechanism is used to explain strain hardening and creep in single crystals.

For polycrystalline plastic behavior, the work of Kunze and of Taylor is reviewed, and a new theory of the author is added, based on an interlocking mechanism between the crystals. The external force which a given crystalline structure withstands at a certain deformation is considered to have two components: (1) The force necessary to deform the crystals, assuming that they are independent of each other; and (2) a transverse force of the same sense associated with the interlocking and interaction of the crystals. The effect then is to produce in polycrystals a state of stress approaching hydrostatic tension or compression, which would not occur in single crystals. The mechanism can then be used to predict polycrystalline behavior from single-crystal data.

Louis F. Coffin, Jr., USA

832. P. W. Bridgman, "Large plastic flow and the collapse of hollow cylinders," *J. appl. Phys.*, Mar. 1948, vol. 19, pp. 302–305.

Experiments are described on the collapse by inward radial flow of hollow steel cylinders which are closed at both ends and subjected to hydrostatic pressure over the entire external surface. For certain specimens the flow was such as to result in almost complete closing of the internal bore, with enormous distortion of the original grains in its immediate vicinity. The theoretical treatment (briefly indicated) is purely plastic and it is assumed that there are negligible changes in the volume and length of the specimens, these assumptions and neglect of elastic deformations being justified by the experimental results.

If it be assumed that there is no strain hardening and that plastic flow is determined by the maximum shear stress criterion, then analysis shows that $P/\log(r_o/r_i)$ = the critical shear stress, where P is the collapsing pressure and r_o and r_i are the final external and internal radii respectively. Actually the experimental results show that $P/\log(r_o/r_i)$ continually increases with P even into regions of deformation where the granular structure dominates, and the above criterion for plastic flow is therefore incorrect, and there is strain hardening. A theoretical deduction of the strain-hardening curve is possible from the experimental results. Only approximate information may be derived, but it is definitely indicated that the strain hardening is at a rate only $^1/_2$ to $^1/_3$, or less, of that for ordinary tension tests.

H. G. Hopkins, England

833. J. E. Dorn and E. G. Thomsen, "The ductility of metals under general conditions of stress and strain," *Trans. Amer. Soc. Metals*, 1947, vol. 39, pp. 741–772.

A method for calculating the limiting strains at fracture is presented in this paper. It is assumed that the metal is homogeneous, isotropic, and that the volume remains constant. The authors formulate the stress-strain relations for the plastic state in terms of the deviations of the stress and infinitesimal strain tensors. These are then combined with Hooke's law to obtain the combined elasto-plastic stress-strain relations.

The authors next compare their experimental data as well as that of other experimenters with several proposed laws of fracture. Evidence seems to favor the maximum shear stress law, although it has some shortcomings.

The finite strains at rupture are obtained by integrating the infinitesimal strains over the whole stress history. The principal strain components are predicted and compared with experimental results for various laws of fracture. Again the maximum shear

stress law indicates good agreement, and in certain instances the maximum normal stress law also seems to apply with good accuracy.

Paul F. Chenea, USA

834. **V. G. Berezanzev, "Limit equilibrium of a medium having internal friction and cohesion in a stressed state, symmetrical to the axis"** (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), Jan.-Feb. 1948, vol. 12, pp. 95-100.

This paper treats stresses in a medium with internal friction and cohesion, stressed symmetrically to the axis and without body forces. The general theory of principal stresses and lines of rupture is applied to the case of a cylindrical excavation with uniformly distributed radial pressure on the vertical cylindrical surface of the excavation and a continuous load on the horizontal surface of the medium. Then for uniformly distributed loading on the horizontal surface of the medium due to pressure with a cylindrical stamp, the lines of rupture and the pressure of the stamp are approximately calculated.

Z. Bažant, Czechoslovakia

835. **M. J. Druyvesteyn, "Experiments on the effect of low temperature on some plastic properties of metals,"** *Appl. sci. Res. Sec. A*, 1947, vol. 1, no. 1, pp. 66-80.

The yield strength, the ultimate strength, the elongation, and the hardness of a number of pure polycrystalline metals were measured by the author at room temperature and at −183 C. The changes which occurred in these properties are compared by taking into consideration the crystal structure of the metals.

The increase in the yield strength at −183 C is greater for the body-centered metals and for zinc, cadmium, and tin than for the face-centered metals. This larger increase in the yield strength was accompanied by a greater tendency toward brittleness in the body-centered metals. In all the metals the hardness and the ultimate strength increased with decreasing temperature.

The ratios of properties at the two temperatures are given in tabular form in the paper.

Evan A. Davis, USA

836. **F. A. Bakhshiyan, "The viscous-plastic flow at the impact of a cylinder on a plate"** (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), Jan.-Feb. 1948, vol. 12, pp. 47-52.

The paper presents a highly speculative treatment of the problem of stresses induced in a plate, struck transversely by the flat end of a cylinder moving longitudinally. Shearing stresses on concentric cylindrical surfaces in the plate are assumed to consist of a constant plastic part, and of a viscous part the same as in a viscous fluid. A very involved solution of the differential equation is carried out, assuming a constant extension of the viscous region and disregarding all elastic stresses and deformations as well as variations of stresses through the thickness of the plate.

A. Hrennikoff, Canada

837. **Noah A. Kahn and Emil A. Imbembo, "A method of evaluating transition from shear to cleavage failure in ship plate and its correlation with large-scale plate tests,"** *Weld. Res. Suppl.*, Apr. 1948, vol. 13, pp. 169-182.

A laboratory test is described which is capable of evaluating the susceptibility of ship plate to brittle or cleavage-type fracture. The method utilizes a notched specimen 3 in. × 5 in. × the full plate thickness.

The specimen is loaded asymmetrically in static tension to complete failure. The load-deflection diagram indicates the energy input required to start initial failure and to propagate the fracture to completion. It was found that the mode of fracture may be correlated with the energy input to propagate the fracture to completion. The method is shown to be capable of sharply defining the location of the transition temperature range of ship plate from ductile to brittle-type fracture.

R. L. Bisplinghoff, USA

Design Factors, Meaning of Material Tests

(See Revs. 817, 822, 834)

Material Test Techniques

(See also Revs. 778, 789, 795, 817, 837)

838. **W. A. Crouse, D. C. Caudill, and F. W. Reinhart, "Effect of simulated service conditions on plastics during accelerated and two-year weathering tests,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1438, May 1948, pp. 1-8.

No correlation was found between the results of two-year outdoor weathering and accelerated laboratory tests (Federal Specification L-P-406a) on 11 laminates, having bakelite, marco, allymer, and lignin binders.

Hans F. Winterkorn, USA

839. **Paul R. Shepler, "Explosive impact tests,"** *Proc. Soc. exp. Stress Anal.*, vol. 5, no. 1, pp. 1-25.

The use of an explosive charge enables tensile impact tests to be made at very high straining speeds. The true stress at fracture, a quantity little used previously, is compared over a range of speeds for a number of materials.

The section in the appendix dealing with corrections to the load at fracture seems to be oversimplified.

A. F. C. Brown, England

Mechanical Properties of Specific Materials

(See also Revs. 819, 822, 835, 838, 839)

840. **D. T. Berglund, "Rigidity of gelatin (Fasthetsbestämning av geler),"** *Tekn. Tidskr.*, Sept. 13, 1947, vol. 77, pp. 671-676.

The author makes a critical study of the various methods which have been used for determining the rigidity of gelatin and pectin products. The fitness of the different methods is discussed from a consideration both of fundamentals and of detail problems. The author's idea is that the most valuable methods are those which are adapted to give the ultimate strength of the material tested. The article includes a very extensive literature index.

Ragnar Nilson, Sweden

841. **C. Gurney and S. Pearson, "Fatigue of mineral glass under static and cyclic loading,"** *Proc. roy. Soc. Lond. Ser. A*, Mar. 18, 1948, vol. 192, pp. 537-544.

Two sets of experiments were conducted to investigate the effect of cyclic loading on the fatigue strength of round glass rods. In both sets four-point loading rotating-bend fatigue machines were used. In the first set the time to fracture under constant-stress cycle was measured at three frequencies of cyclic bending stress: zero (that is, static loading), 14 cpm, and 10,000 cpm. In the second set the cyclic bending stress was increased at a constant rate until fracture, at the same three frequencies as above.

Comparison of the data shows little difference in the times to

fracture when the test pieces are stationary or are rotated at 14 or 10,000 rpm. It is concluded that the primary fatigue effect is independent of the number of cycles to fracture, and that the nature of cyclic fatigue in glass is quite different from that in metals. This conclusion is explained on the basis that the cyclic fatigue in glass is caused by the spreading of cracks under the action of tension stress and is thus a manifestation of the same phenomena as "fatigue" under static loading. R. L. Bisplinghoff, USA

842. James A. Miller, "Stress-strain and elongation graphs for Alclad aluminum-alloys 24S-T and 24S-T81 sheets," *Nat. adv. Comm. Aero. tech. Note*, nos. 1512 and 1513, May 1948, pp. 1–37 and 1–36.

These reports present tensile and compressive stress-strain and elongation data for clad aluminum-alloys 24S-T and 24S-T81 sheets, for longitudinal and transverse directions, plotted on a dimensionless basis to make them applicable to material having yield strengths different from those of the test specimens.

John E. Goldberg, USA

843. L. Seigle and R. M. Brick, "Mechanical properties of metals at low temperatures; a survey," *Trans. Amer. Soc. Metals*, 1948, vol. 40, pp. 813–869.

A review is presented of the ductility characteristics of various metals at subatmospheric temperatures. The authors conclude that only metals having a face-centered cubic structure retain ductility at low temperatures, and that the brittleness of all other structures, including body-centered cubic ferritic steels, probably is related to the relative slip, twinning, and cleavage mechanisms of single crystals. T. J. Dolan, USA

844. G. Welter, "The effect of notching on materials of construction under static and dynamic tension," *Metallurgia*, Sept. 1947, vol. 36, pp. 283–286; Oct. 1947, vol. 36, pp. 339–343; Nov. 1947, vol. 37, pp. 36–40.

Static and impact tests were made on aluminum, copper, brass, steel, and several of the more common alloys. The maximum striking speed for the impact tests was 29 fps. A load elongation diagram was automatically recorded. The active gage length of the cylindrical specimens was varied from $^1/_{32}$ in. to 2 in. Specimens containing 90 and 160-deg V-notches were also tested.

The total energy and the energy per unit volume for both the static and the dynamic tests were obtained as functions of the gage length. The ultimate strengths in the static tests were also plotted as functions of the gage length. The effect of the speed of loading on the ultimate strength and on the elongation of the 2-in. gage lengths was determined.

The dynamic as well as the static strength increased with decreasing gage length. The strength increased with the testing speed. The strength of the more ductile materials was increased by the introduction of a notch. Evan A. Davis, USA

845. E. E. Reynolds, J. W. Freeman, and A. E. White, "Evaluation of two high-carbon precision-cast alloys at 1700 and 1800 F by the rupture test," *Nat. adv. Comm. Aero. tech. Note*, no. 1130, Sept. 1946 (publ. in 1947), pp. 1–26.

Data obtained from rupture tests are presented to show the performance of two precision-cast alloys at 1700 and 1800 F. One alloy, the NT-2 type, is a high-carbon Ni-Cr-Co-Mo-W-Ta alloy, being essentially a 30 per cent nickel modification of alloy N155, with 1 per cent carbon and the columbium replaced by 2 per cent tantalum. The other alloy, the VT2-2 type, is similar to Vitallium but modified with 1.2 per cent carbon and 2 per cent tantalum. The two alloys were previously developed for the Bureau of Ships, U. S. Navy Department.

The results indicate that the NT-2-type alloy is stronger than the VT2-2 type for time periods longer than 10 hr. The rupture strengths of the NT-2 type compare favorably with those of X-40, which was the best alloy previously tested at 1700 and 1800 F. The modified Vitallium alloy was superior in rupture strength to the standard alloy of this type for time periods up to about 1000 hr.

Harry A. Williams, USA

846. Åke Holmberg, "Yield point and ultimate stress of hot-rolled steel rods (Sträck- och brottgräns hos varmvalsade rundjärn)," *Tekn. Tidskr.*, Sept. 27, 1947, vol. 77, pp. 724–725.

The author has studied the size-effect of the cross-section diameter of hot-rolled steel rod on some of the properties which are conventionally used in structural engineering in Sweden. The size-effect is particularly pronounced in connection with variations of the yield point and the ultimate stress and the relation between them. The author's conclusions are in conformity with those which have been published by other research workers.

Ragnar Nilson, Sweden

Mechanics of Forming and Cutting Processes

(See also Rev. 834)

847. R. Hill, "A theoretical analysis of the stresses and strains in extrusion and piercing," *J. Iron Steel Inst.*, Feb. 1948, vol. 158, pp. 177–185.

This paper presents a theoretical analysis of the stresses, strains, and expenditure of work in certain extrusion and piercing processes. The analysis is two dimensional, although in some instances the results may be applied quantitatively to axially symmetrical conditions.

The basis for the analysis is the theory of orthogonal slip lines as formulated by H. Hencky and H. Geiringer, in which the material is assumed to obey the yield criterion of constant maximum shear throughout the plastic region. The method of solution is one of trial and error.

The theory is applied in detail to inverted two-dimensional extrusion with 50 per cent reduction, and the distortion, from passing through the die, of a square grid ruled on the billet is obtained.

The author discusses extrusion with other reduction ratios, square dies, wedge-shaped dies, and piercing with a flat punch. Calculated values of die and extrusion pressures for various reduction ratios are presented. Paul F. Chenea, USA

848. G. D. S. MacLellan, "A critical survey of wire-drawing theory," *J. Iron Steel Inst.*, Mar. 1948, vol. 158, pp. 347–356.

The author presents a survey and analysis of important past researches in wire drawing, dating from 1915 to the present. The discussion centers principally around the drawing force required as a function of reduction of area, coefficient of friction, and die angle. The underlying theories adopted and results obtained in a number of publications are discussed in a critical manner and compared.

It is pointed out that in all of the past analyses it is necessary to either assume or infer a value for the coefficient of friction. No direct method for its determination has been developed. The author, however, suggests two possible approaches to a more

direct determination of coefficient of friction, one based on measurements of the splitting force exerted on the die and the other based on tests with back pull on the wire.

Thirty-one British, American, German, and Japanese references relating to wire drawing are listed. William Schroeder, USA

849. M. C. Shaw, "Mechanical activation—a newly developed chemical process," *J. appl. Mech.*, Mar. 1948, vol. 15, pp. 37–44.

The paper deals with a new chemical process, called "mechanical activation," developed by the author. Many reactions between metals and organic reagents—called "organometallic" reactions—are difficult to initiate and control by conventional methods. The author shows that when the metal involved in the reaction is cut at a suitable speed in the presence of the organic reactant (which takes the place of the cutting fluid), many of the above-mentioned reactions can be easily initiated, continuously maintained, and controlled. The new process utilizes the high local pressure, temperature, and the nascent highly stressed metal surface of the chips existing during the cutting process.

The metal-cutting principles are first discussed, pointing out the role of the cutting fluid in eliminating the built-up edge and reducing friction between the tool and the metal being cut. A chemical reaction takes place between the cutting fluid and the nascent metal chip, thus severing the connection between the metal being cut and the chip. With certain reagents as cutting fluid this chemical action becomes autocatalytic and may be used in the synthesis of certain chemical products.

After explaining the conventional methods used today in initiating organometallic reactions with the use of Grignard reagents, the author describes mechanical activation and the apparatus in which the experiments were carried out. In conclusion, he points out that the time required to initiate these reactions is much shorter, the rate of production is continuous and greater, and the control is easier in the new process than it is in the conventional method. Nicholas Sag, Australia

Soil Mechanics; Seepage

(See also Revs. 783, 858, 902)

850. W. H. Smith, G. P. Tschebotarioff, E. R. Ward, J. R. Bayliss, P. P. Brown, H. Epstein, L. A. Palmer, J. C. Gebhard, and L. C. Coxe, "Lateral earth pressures on flexible retaining walls," *Proc. Amer. Soc. civ. Engrs.*, Jan. 1948, vol. 74, pp. 3–100.

This symposium presents results, analysis, and practical applications of the first major American research on lateral earth pressures to be published in many years. A model bulkhead 12-ft high × 18 ft long was built and tested at Princeton University under Tschebotarioff's supervision, in co-operation with the Bureau of Yards and Docks, U. S. Navy Department.

Experimental studies were made of the amount and distribution of lateral pressure caused by a liquid-clay and by a sand-clay mixture, of the reductions in these pressures that can be obtained by introducing a sand dike between bulkhead and fill, and of the effect of arching of sand on the distribution of lateral pressure. Further tests are now in progress, with the same and supplementary apparatus, to study the behavior of other soils, the lateral pressures under prototype intensities of vertical pressure, the effect of wall rigidity, and the nature of restraint due to friction on the bottom of the test vessel or due to horizontal discontinuities in prototype soils.

Bending strains of the bulkhead were measured at about 100 points using electric-resistivity strain gages. Horizontal reactions at anchor and at toe were measured, as were the total vertical reaction of the wall and the moment at the toe. Horizontal deflections were measured at about 70 points. Experimental procedures seem to have utilized the best scientific techniques, as well as some new methods, and the test results seem to have been properly observed and analyzed. Checks on accuracy are very good.

Some of the major findings of the model tests are as follows: (1) A sand dike behind the bulkhead effectively absorbs a large portion of the lateral pressure of a liquid-clay backfill; this is due to structural influence rather than to drainage. (2) Conventional methods of estimating lateral pressures on the basis of laboratory test results give considerably lower values than those measured on the model. (3) No decrease of bulkhead bending moments was observed due to arching of sand when placed in uniform layers. (4) A relatively rigid layer underlying the backfill can serve, through friction, to reduce lateral pressures on the lower portion of the bulkhead.

The contributions by Epstein, Palmer, Gebhard, and Coxe consider design and construction matters, indicating applications of the test results to design, and examples of successful and unsuccessful quay wall designs and constructions. It is pointed out that there are no known failures of bulkheads which can be attributed to failures of steel sheet piles, due, apparently, to the fact that the structural members are usually designed for a much higher factor of safety than that commonly allowed for the soil. C. Martin Duke, USA

851. M. A. Lookomskaya, "Solution of some problems on flow of a liquid to wells" (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, Nov.-Dec. 1947, vol. 11, pp. 621–628.

The paper gives some solutions for the plane flow of a perfect incompressible liquid toward a well through soil regions G, G_1, G_2, ... of different permeability or through soil regions filled with liquids of different viscosity, such as water and oil.

Three special cases are considered. In the first case the well is located in a plane strip G_1 adjacent to a half plane G_2. In the second case the well is located in a strip G separated by two parallel lines from two half planes, G_1 and G_2. In the third case the well is located within a circle G_1, G_2 being another circle exterior to G_1, and G the remainder of the flow plane.

The physical problems are reduced to the mathematical problems of solving the functional equations

$$w_1(z) - \overline{w_1(z^*)} = w_2(z) - \overline{w_2(z^*)}$$

$$\frac{w_1(z) + \overline{w_1(z^*)}}{c_1} = \frac{w_2(z) + \overline{w_2(z^*)}}{c_2}$$

where w is the complex potential of the flow, c_1 and c_2 are distinct constants, and we have respectively for each of the three cases

$$z^* = \bar{z};\ z^* = \bar{z} - 2h;\ z^* = C + R^2/\overline{z - C}$$

where a bar denotes the conjugate, and h, C, and R are geometric parameters. The solutions of these equations are then given for each of these cases. R. Bellman, USA

852. J. Jáky, "Stability of earthworks in the plastic state, I. (Sur la stabilité des masses de terre complètement plastiques, I.)," *Publ. tech. Univ. Budapest (Müegyetemi Közl.)*, 1947, no. 2, pp. 129–151.

In earthworks made of clay of a water content above the plastic limit, the angle of internal friction is very small and stability is assured only by the cohesion. Stability and internal stress problems of earthworks in a plastic state are—apart from the approximate solution of L. Jurgenson—still unsolved questions of soil

mechanics, and an exact solution is needed, particularly for the stability of high earth dams.

Using the laws of plasticity, the author proves that the slip lines are circles, as has long been supposed and assumed by Swedish and American engineers in the practical solution of stability problems. All other solutions are—according to the author's view—incorrect and the so-called squeezing test is also erroneous because Prandtl's solution is not applicable to specimens having symmetrical axes. For the proper knowledge of slip lines and stresses the author gives a solution for the compression problems of highly plastic clay soils; active and passive earth-pressure values, the stability of earth slopes, and the maximum bearing values for mild clayey subsoils are detailed and illustrated with numerical examples.

It would be highly desirable if the results obtained could be illustrated by practical tests. Ch. Széchy, Hungary

853. Karl Terzaghi and Ralph B. Peck, "Soil mechanics in engineering practice," John Wiley and Sons, New York, N. Y., 1948. Cloth, 6 × 9.3 in., 566 pp., 218 figs., \$5.50.

The book is divided into three parts. The first part (137 pages) deals with the physical properties of soils and discusses classification tests, consolidation, permeability, and shear tests of soils, as well as other related topics.

The second part (118 pages) deals with theoretical soil mechanics and gives the fundamental conventional assumptions and the procedures for the analysis of problems of plastic equilibrium in soils, including lateral earth pressures, bearing capacity of footings, and stability of slopes. It also deals with problems of settlement and consolidation, seepage, and piping.

The third part (282 pages) deals with problems of design and construction of retaining walls, open cuts, earth dams, and footings, pile, pier, and dam foundations. A considerable portion of this part is devoted to the methods and scope of soil exploration programs. Some special problems, such as the compaction of soils and the effects of vibrations, are briefly outlined.

Subgrade studies for highways and airports, sheet-pile bulkheads, stability and stiffness of cofferdams, earth tunneling, culvert design, and grouting with cement and chemicals are not treated in the text but five pages of the appendix give some references to these topics.

The book contains much valuable information concerning the theory and the practical applications of Soil Mechanics, a science which is generally recognized to have been founded by the senior author. Gregory P. Tschebotarioff, USA

Potential or Laminar Incompressible Flow

(See also Revs. 877, 894, 902)

854. L. I. G. Kovasznay, "Laminar flow behind a two-dimensional grid," *Proc. Camb. phil. Soc.*, Jan. 1948, vol. 44, pp. 58–62.

The exact equations for laminar flow are difficult to solve because of the presence of nonlinear terms. The author has succeeded in finding a useful solution in which these nonlinear terms vanish identically. This solution consists of a uniform velocity plus disturbance velocities that are periodic in the dimension perpendicular to the uniform velocity and vary exponentially with the dimension parallel to the uniform velocity. The solution contains a parameter which must satisfy a quadratic equation containing the Reynolds number based on the uniform velocity and the length of periodicity.

The solution corresponding to one root can be interpreted as the diffusing laminar wakes behind a two-dimensional grid. The solution corresponding to the other root is a reversed flow along similar streamlines, except that the pattern is greatly contracted in the direction parallel to the main flow. This contraction factor is unity for zero Reynolds number, and increases rapidly for Reynolds numbers above one.

These exact solutions should prove quite useful in studying the characteristics of laminar wakes behind grids, particularly the stability of such wakes. Francis H. Clauser, USA

855. P. Neményi and R. Prim, "Some properties of rotational flow of a perfect gas," *Proc. nat. Acad. Sci. Wash.*, Mar. 1948, vol. 34, pp. 119–124.

The authors develop several theorems concerning the steady rotational motion of a frictionless perfect gas in two dimensions without body forces. It is shown that: (*a*) The speed is constant along streamlines only if they are concentric circles or parallel lines; (*b*) constant vorticity along a streamline implies constant speed; (*c*) the coincidence of curves of constant vorticity and constant speed implies that they coincide with the streamlines, which will be concentric circles or parallel lines.

Another proposition is developed concerning the type of orthogonal cylindrical co-ordinates (for which in the x,y plane a net of square elements is formed, i.e., $g_{11} = g_{22}$) for which all three reduced velocity components depend on only one co-ordinate.

Flow fields are also discussed for which $v_z = 0$, $\mathrm{curl}_z\bar{v} = 0$. Such fields can be constructed from a function $\varphi(x,y,z)$ which is a potential function as far as v_x and v_y are concerned ($v_x = \varphi_x$, $v_y = \varphi_y$) but not for v_z. We thus have potential flow in any plane parallel to the x,y plane, but the flow field will vary in the z direction, while $v_z = 0$. Stewart Way, USA

856. Carl Eckart, "The theory of the anelastic fluid," *Rev. mod. Phys.*, Jan. 1948, vol. 20, pp. 232–235.

The general theory of anelastic substances proposed in an earlier paper by the author [*Phys. Rev.*, Feb. 15, 1948, vol. 73, p. 373] is simplified for the case of an anelastic fluid. In this simplification it is supposed that the fluid density departs only slightly from a constant value ρ_0, so that the condensation $s = (\rho - \rho_0)/\rho_0$ is small. Then the relation between the condensation s and the sum of the principal strain components σ, which for a purely elastic substance is simply $\sigma + s = 0$, becomes

$$\frac{\partial}{\partial t}(\sigma + s) = -A(\lambda_1\sigma + \lambda_2 S)$$

and the equation implied by the conservation of momentum of the fluid with velocity $\mathbf{u}$ becomes

$$\rho_0\frac{\partial \mathbf{u}}{\partial t} = -\nabla P + N\nabla^2\mathbf{u} + (N + N')\nabla\nabla\cdot\mathbf{u}$$

where $P = (\lambda_3 - \lambda_2)s + (\lambda_2 - \lambda_1)\sigma$ is the total elastic pressure.

In these equations A is a coefficient of anelasticity of the fluid, N and N' are coefficients of viscosity, such that $N > 0$, $3N' + 2N > 0$; $\lambda_1, \lambda_2, \lambda_3$ are constants such that $\lambda_1 > 0$, $\lambda_1\lambda_3 > \lambda_2^2$. As a consequence, it is shown that the slow steady motion of an anelastic fluid obeys the same laws as are formulated in the Navier-Stokes' theory of a viscous fluid. However, the Reynolds number criterion for the validity of the approximations involved in this theory is shown to be merely necessary and not sufficient.

On the other hand, new phenomena are exhibited by the equations for the propagation and dissipation of longitudinal waves. These include hysteresis in the response of density to the pressure, which vanishes for very high and very low frequencies. The related velocity of the propagation is a complex number de-

pending on the frequency. Two relaxation times are defined corresponding to the zeros of the velocity of propagation. One of these is associated with the hysteresis, and the other with viscosity.

Robert C. F. Bartels, USA

857. G. F. Carrier and C. C. Lin, "On the nature of the boundary layer near the leading edge of a flat plate," *Quart. appl. Math.*, Apr. 1948, vol. 6, pp. 63–68.

The authors develop two methods of solution of this problem. Their general procedure is to neglect the nonlinear terms in the partial differential equation for the stream function ψ, and obtain the solution of the biharmonic equation as the zero-order approximation which represents the Stokes'-type slow flow. The first approximation is to solve for ψ from the equation with the zero-order solution inserted in its nonlinear terms. Generalization of the approximation to the nth order is then obvious.

First, polar co-ordinates r, θ are used and the proper Stokes' slow flow is given. Higher approximations are convergent within some circle around the leading edge of the plate. With the exception of a term in $\ln r$, the solution coincides with the Blasius solution for small values of θ, and the constant of integration is thereby determined.

In the second method the biharmonic equation yields the former Stokes' slow flow for η small and irrespective of ξ, where ξ and η are parabolic co-ordinates. If ξ is large while η is of the order of unity, a modified Blasius solution with η as its independent variable is derived within the region $0 \leqq \eta \leqq \epsilon\xi$, where ϵ is small. There is an overlap of the regions of validity, where the two solutions agree.

P. Y. Chou, China

858. Jean Ferrandon, "The laws of flow in filtration (Les lois de l'écoulement de filtration)," *Génie civ.*, Jan. 15, 1948, vol. 125, pp. 24–28.

The author develops a very general theory of the flow through pervious materials. Two coefficients are introduced: (1) A discharge per unit area (or velocity) k; (2) a pure number μ measuring the net area of the cross sections of the interstices. The material is assumed to consist of small corpuscles, each one penetrated by tiny tubes in any direction, so that k and μ have to be considered at any point as functions of the directions.

This is shown to be conducive to the establishment of two symmetric matrices of second order

$$\overline{\overline{K}} = \text{matrix of permeability}$$
$$\overline{\overline{s}} = \text{matrix of free cross sections}$$

They are defined by the symbolic formulas giving their elements

$$\overline{\overline{K}}_{ij} = \tfrac{1}{2}\int\int k\mu\alpha_i\alpha_j d\sigma; \quad \overline{\overline{s}}_{ij} = \tfrac{1}{2}\int\int \mu\alpha_i\alpha_j d\sigma$$

The α's denote the direction cosines and the integrals are extended over the unit sphere surrounding the point considered. $\overline{\overline{K}}$ and $\overline{\overline{s}}$ are, in general, functions of time and space.

Introducing, as is usual in percolation problems, the potential of flow in the sense of Bernoulli

$$\varphi = \frac{p}{\gamma} + z$$

the author condenses the description of the velocity field and the field of the hydrodynamical forces into the formulas (discharge percolating across the unit volume) $= -\text{div}(\overline{\overline{K}} \cdot \text{grad } \varphi)$; and (force exerted by the flow upon the same unit volume) $= -\gamma \cdot \overline{\overline{s}} \cdot \text{grad } \varphi$.

This conception is worked out for several cases, but it is difficult to judge its practical value since no method or example is given to show how to connect the coefficients k and μ and the matrices $\overline{\overline{K}}$ and $\overline{\overline{s}}$ with physical experience.

Wilhelm Spannhake, USA

859. J. Kampé de Fériet, "Harmonic analysis of the two-dimensional flow of an incompressible viscous fluid," *Quart. appl. Math.*, Apr. 1948, vol. 6, pp. 1–13.

The author considers two-dimensional flow of an incompressible viscous fluid in a bounded domain D, the velocity vanishing at the boundary B. He introduces the Fourier transform of the vorticity $Z(\omega_1,\omega_2,t)$ (ω_1 and ω_2 being real) and the Fourier transforms of the stream function and of the velocity components. He also introduces the spectral function $\gamma(\omega_1,\omega_2,t)$ of the kinetic energy E of the fluid, defined by $E = \int\int \gamma(\omega_1,\omega_2,t)\, d\omega_1 d\omega_2$ (the integration being carried out over the whole plane ω_1,ω_2). He proves that the Fourier transforms of the stream function and of the velocity components are expressed in terms of Z; these transforms are equal to the products of Z by very simple rational functions of ω_1 and ω_2. He then proves that the spectral function has also a very simple expression in Z. To prove these results only the boundary conditions (and not the equation of motion) are utilized, so that the results are valid whether the fluid is viscous or not.

It is known that, for a viscous incompressible fluid, the kinetic energy E is a function of t, decreasing more rapidly than an exponential function of the time in the form $E_0e^{\alpha t}$. Utilizing this inequality satisfied by E, the author establishes similar inequalities which are satisfied by the functions Z and γ. He then deduces from the equation of motion an integro-differential equation satisfied by Z, "the study (of which) seems to be very difficult." This equation contains a term Φ, the calculation of which necessitates the knowledge of the vorticity and of its normal derivative on the boundary B.

Finally, the author supposes that the domain is no longer bounded but is the entire plane with the velocity vanishing at infinity. If the velocity components and the vorticity decrease with sufficient rapidity at infinity, the term Φ will vanish and the integro-differential equation is simplified.

Ratip Berker, Turkey

860. I. E. Garrick, "On the plane potential flow past a lattice of arbitrary airfoils," *Nat. adv. Comm. Aero. Rep.*, no. 788, 1944 (issued in 1948), pp. 1–16.

The two-dimensional incompressible potential flow past a lattice airfoil of an arbitrary shape is investigated theoretically. A transformation is developed to take a simple lattice of staggered line segments into a circle. The fore-and-aft infinite regions of the lattice configurations map into a pair of symmetrical points outside the circle on the axis of reals. The transformation takes a lattice of airfoils of arbitrary section into a near circle which is then first transformed into a circle without control on the singularities corresponding to the infinities, and then into a circle with the infinities mapping as described above. The flow pattern and pressure distribution over the airfoil section may then be determined.

An outline of the procedure is given for calculating pressure distributions. This may be followed without reference to the theory. Several illustrative pressure-distribution examples for the NACA 4412 section are presented and compared with the simple airfoil case.

M. G. Scherberg, USA

861. Max Shiffman, "On free boundaries of an ideal fluid," *Commun. appl. Math.*, Jan. 1948, vol. 1, pp. 89–99.

This paper describes a method of determining the stationary

irrotational flow in two dimensions of an ideal incompressible fluid which is bounded partly by free boundaries. This method differs from that initiated by Helmholtz and elaborated considerably by others.

The new method deals directly with the actual physical plane of the flow. It consists in extending the analytic function describing the flow across the free boundary and finding the boundaries and singularities of this extension. The extended flow is called the image of the actual flow.

The author expresses the belief that the new method provides a clearer physical picture of the formation of the free boundary. Since the problem is reduced to one involving fixed walls alone it is solvable by conformal mapping without the necessity of additional integration.

The general problem of flow in two dimensions is formulated in the conventional manner in terms of analytic functions of a complex variable. Using the Schwarz principle of reflection, the author develops expressions for the images of a streamline, a corner, a stagnation point, a source, a vortex, and a constant flow at infinity.

The method is applied to several simple examples such as a jet, a Borda mouthpiece, and flow about a plane lamina.

W. E. Wilson, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 857, 869, 882, 883, 900, 909)

**862. Raymond Goethals, "Digest of experimental results concerning turbulent boundary layers (Condensation des résultats expérimentaux concernant les couches limites turbulentes)," ** *C. R. Acad. Sci. Paris*, Mar. 31, 1948, vol. 226, pp. 1073–1075.

Certain correlations regarding turbulent boundary-layer velocity distributions obtained from work of the author on diffusers, flow through pipes, and along flat plates are examined with a view to improving existing semiempirical formulas. The accepted logarithmic variation is confirmed, while boundary-layer development in a diffuser or along a flat plate is not considered.

Newman A. Hall, USA

863. G. K. Batchelor, "Kolmogoroff's theory of locally isotropic turbulence," *Proc. Camb. phil. Soc.*, Oct. 1947, vol. 43, pp. 533–559.

This is a detailed critical presentation of the new theory of "locally isotropic" turbulence, due to A. N. Kolmogoroff [*C. R. Acad. Sci. URSS*, 1941, vol. 30, p. 301; vol. 32, p. 16]. The author has also worked out some of the ramifications of the new theory as applied primarily to turbulence which is both "locally isotropic" and isotropic in the sense used by Taylor and von Kármán-Howarth.

Kolmogoroff's theory presupposes the existence of steady isotropy within some small space-time domain, irrespective of the condition of the over-all turbulence, provided only that the Reynolds number be sufficiently large. The most convenient kinematic variables are differences between velocity components at two points, P and P', rather than the velocities themselves. Of dominant interest are the two primary double-correlation functions, $B_{dd}(r) = \overline{(u'_d - u_d)^2}$, and $B_{nn}(r) = \overline{(u'_n - u_n)^2}$ where u_d and u_n are velocity components parallel and normal, respectively, to the line PP' of length r, u being velocity at P, u' at P'. Triple- and higher-order correlations are defined analogously.

Having postulated "local isotropy," the theory is based upon two similarity hypotheses: (1) The probability distributions defining the turbulence are uniquely determined by the kinematic viscosity ν and the average total rate of energy dissipation (to heat) per unit mass of fluid ϵ. (2) For eddies large compared to the smallest (dissipative) eddies, but smaller than the domain of local isotropy, the probability distributions defining the turbulence depend only on ϵ.

An immediate consequence of the first similarity hypothesis is the parabolic variation of $B_{dd}(r)$ and $B_{nn}(r)$ in the neighborhood of the origin. This is analogous to the well-known behavior of the corresponding functions in ordinary isotropic turbulence; for turbulence which is both isotropic and locally isotropic, the relations are equivalent.

The first and second hypotheses together lead to the prediction of a $^2/_3$-power variation of the correlation function $B_{dd}(r)$ for large r. With the assumption of constant "skewness" and neglecting viscous dissipation, both for large r, a $^2/_3$-power correlation function is also deduced from the dynamical equation of the double and triple correlations.

Since a comparison of this prediction with usual correlation curves gives indecisive results, due apparently to the low Reynolds number of the experiments, the author has worked out additional implications of the theory, which may also be compared with existing experimental results. These predictions, relating to the behavior of the correlation functions, turbulence Reynolds number, and skewness factor, show varying degrees of agreement with experiment. The agreement is good enough to demand new experiments at much higher Reynolds numbers. Following Kolmogoroff, the author has avoided discussion of the prediction of the turbulent energy spectrum.

This paper will be particularly welcome to those who have attempted to fill the gaps in Kolmogoroff's published abstracts.

Stanley Corrsin, USA

864. F. N. Frenkiel, "On third-order correlation and vorticity in isotropic turbulence," *Quart. appl. Math.*, Apr. 1948, vol. 6, pp. 86–90.

Using the series expansion of the double-correlation function $f(r,t)$, including the term in r^4, the author has derived an expression for the basic triple correlation $h(r,t)$, as a function of $f''(0,t)$, $f^{IV}(0,t)$, $N_\lambda(t) = \lambda\sqrt{\overline{u'^2}}/\nu$, and r, where the f derivatives are with respect to r. Secondly, with von Kármán's approximate result for the constancy of $\sqrt{\overline{u'^2}}\lambda^2/(L\nu)$ during the decay of "large Reynolds number" turbulence (obtained by neglecting the viscous term in the correlation equation), he also introduces the Loitsianskii theorem for the conservation of the "disturbance moment." These two relations, together with the general energy decay relation, give expressions for the time rate of change of various characteristic functions describing the turbulence.

Finally, the results of the above two analyses are combined to give a more specific and more approximate expression for $h(r,t)$ valid for small r. This leads to relations for the time variation of additional characteristic quantities. A comparison with the predicted results of Batchelor and Townsend (see Rev. 323, APPLIED MECHANICS REVIEWS, Feb. 1948) shows a small difference in the various decay laws.

It should be noted that the assumptions basic to the Loitsianskii theorem, i.e., that h, f, and f' decrease more rapidly than r^{-4} as $r \to \infty$, have apparently never been rationalized.

Stanley Corrsin, USA

865. Pierre Casal, "Energy dissipation in a homogeneous turbulence (Dissipation de l'énergie en turbulence homogène)," *C. R. Acad. Sci. Paris*, Jan. 5, 1948, vol. 226, pp. 57–59.

The author develops the formula $\overline{W} = \mu\overline{\xi^2}$, where $\overline{W}$ is the mean rate of dissipation of energy per unit volume and $\overline{\xi^2}$ is the mean of

the squared length of the vorticity vector. From this he derives $\lambda^2 = 15\overline{u^2}/\overline{\xi^2}$, where λ is Taylor's microscale of turbulence. This equation, according to the author, is a more precise relation between λ, $\overline{u^2}$, and $\overline{\xi^2}$ than Taylor's statement that "λ may roughly be regarded as a measure of the diameters of the smallest eddies which are responsible for the dissipation of energy." It should be pointed out, however, that Taylor also found the above relations [*Proc. roy. Soc. Lond. Ser. A*, 1938, vol. 164, p. 17].

J. V. Wehausen, USA

866. **Theodore Theodorsen and Arthur Regier, "Experiments on drag of revolving disks, cylinders, and streamline rods at high speeds,"** *Nat. adv. Comm. Aero. Rep.*, no. 793, 1944 (issued in 1948), pp. 1–18.

This paper represents a basic study of skin friction on revolving disks, cylinders, and streamline rods, covering a range of Reynolds numbers up to 7,000,000 and Mach numbers as high as 2.7. A thorough review of the von Kármán-Prandtl skin friction theory is presented as a basis for comparison with the experimental results. In general, the agreement between theory and experiment is excellent and the logarithmic relationship is fully confirmed.

The tests on revolving disks show that skin friction is dependent only on the Reynolds number and is independent of the Mach number.

Studies of the effect of roughness show that the transition Reynolds number is affected slightly by particle size and not at all by particle density.

M. J. Thompson, USA

Compressible Flow, Gas Dynamics

(See also Revs. 866, 878, 881, 891, 898, 912)

867. **Stefan Bergman, "Two-dimensional subsonic flows of a compressible fluid and their singularities,"** *Trans. Amer. math. Soc.*, July-Dec. 1947, vol. 62, pp. 452–498.

This paper, highly mathematical in style and content, deals with the theory of two-dimensional compressible-fluid motion in the subsonic case, doing without the Chaplygin assumption $(\rho_0/\rho)\sqrt{1 - M^2} = 1$.

The paper has chiefly to do with the properties of singularities in the various planes of representation, i.e., the physical, hodograph, logarithmic, and pseudologarithmic planes—singularities which in the physical plane may be sources, sinks, doublets, vortexes, etc. Necessary and sufficient conditions are derived for which a hodograph representation possessing certain classes of singularities leads to a flow in the physical plane about a closed contour. It is shown also that the transition from the physical to the hodograph plane represents in the subsonic case a "quasi-conformal" mapping.

Stewart Way, USA

868. **L. K. Kudriashov, "Plane parallel gas flow past an ellipse"** (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), Jan.-Feb. 1947, vol. 11, pp. 119–128.

The method of Nekrasov [*Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), 1944, vol. 8, no. 4] for calculating the velocity distribution around a body in a compressible stream is reviewed and applied to the flow past an ellipse. The Legendre transformation is used to linearize the differential equation for the velocity potential and the Legendre function is expanded in powers of t_∞, where $(\gamma - 1)[1 + {}^1/_2(\gamma - 1)M_\infty{}^2]^2 t_\infty{}^2 = 2M_\infty{}^2$, M_∞ being the free-stream Mach number and γ the ratio of specific heats of the gas. There results a sequence of Poisson equations to solve for the approximation functions. This technique is thus quite analogous to the Janzen-Rayleigh method.

The velocity distribution on the ellipse is calculated to the order of $t_\infty{}^2$. For an ellipse with thickness ratio 0.5 and free-stream Mach number 0.5, the maximum velocity obtained is 1.59 as compared to 1.607 obtained by Hooker [*Rep. Memo. aero. Res. Comm. Lond.*, no. 1684, 1936]. The results obtained hold only for thin ellipses, since in the final expansion only the first terms of certain expansions are retained.

F. Edward Ehlers, USA

869. **K. Oswatitsch and K. Wieghardt, "Theoretical analysis of stationary potential flows and boundary layers at high speed,"** *Nat. adv. Comm. Aero. tech. Memo.*, no. 1189, Apr. 1948, pp. 1–59 (transl. from *Lilienthal Ges. Luftfahrtforsch. Ber.*, S. 13-1 Teil, 1942, pp. 7–24).

In Part I, using mass-flow considerations, the author finds that beyond a critical Mach number symmetrical transonic flow is no longer possible. A method is also derived for computing transonic flow fields. It starts assuming the flow at infinity to be according to the Prandtl-Glauert hypothesis. The continuation of the flow inward toward the body is then found by means of a stepwise numerical process which ends with finding the contour of the boundary streamline.

Part II is, to the reviewer's knowledge, the first theoretical attempt to analyze the interaction of laminar and turbulent boundary layers with the outer supersonic potential flow. The problem treated investigates the effect upon the outer supersonic potential flow, whose velocity is assumed constant, of a small velocity perturbation imposed at a point on a flat plate.

The variation of the boundary-layer displacement thickness with the velocity of the outer stream is first derived by von Kármán's momentum relation for both laminar and turbulent cases. In the laminar case the procedure is essentially that of Pohlhausen; in the turbulent case the displacement thickness is found by using an empirical velocity profile and neglecting the Reynolds stresses in the Navier-Stokes' equation. The perturbation velocity potential is then found as a solution of the Prandtl-Glauert equation satisfying the condition that the surface of the boundary layer is a streamline. The result indicates a tendency for a "catastrophic" thickening of the boundary layer, with a corresponding influence upon the outer flow.

Hideo Yoshihara, USA

870. **T. M. Cherry, "Flow of a compressible fluid about a cylinder,"** *Proc. roy. Soc. Lond. Ser. A*, Dec. 23, 1947, vol. 192, pp. 45–79.

The partial differential equation for compressible flow is linearized by means of a Legendre transformation to independent variables in the hodograph plane. By separation of variables a set of exact solutions is obtained in the form $\Omega_n = q^n F_n(\tau)$, where q is the speed, τ is the squared ratio of the speed to the limiting velocity, and F_n is a hypergeometric function. The author establishes asymptotic forms for both the subsonic and supersonic ranges by the method of steepest descent. A partial fraction expansion of F_n about its poles, considered as a function of the order n, is also derived.

To obtain the corresponding compressible flow from the incompressible solution, Ω_n is substituted in place of q^n in the expansion about the stagnation point of the Legendre field function for incompressible flow. Then the partial fraction expansion for F_n is substituted; the order of summation is interchanged and the

power series is reduced to a closed form. This result furnishes a means of obtaining the analytic continuation of the Legendre field function into the various regions in the hodograph plane. The results hold only for irrotational flows.

Relations for finding the co-ordinates in the physical plane and the associated stream function are given. The expansions of the stream function are in terms of a different hypergeometric function, for which the author obtains a partial fraction expansion. Explicit relations for the flow around a circular cylinder are obtained.

The author also gives a short critical analysis of the failings of the method of analytic continuation given by Tsien and Kuo. The author's method is essentially the same as that given by Lighthill [*Proc. roy. Soc. Lond. Ser. A*, 1947, vol. 191, p. 323]. The asymptotic forms of the hypergeometric functions given by the author can be shown to be equivalent to those of Tsien and Kuo [*Nat. adv. Comm. Aero. tech. Note*, no. 995, 1946].

F. Edward Ehlers, USA

871. F. Shultz-Grunow, "Gas-dynamic investigations of the pulsejet tube. Parts I and II," *Nat. adv. Comm. Aero. tech. Memo.*, no. 1131, Feb. 1947, pp. 1–65 and 1–20 (transl. from *Inst. Mech. T. H. Aachen Forschungsber.*, no. 2015).

In Part I the author gives the fundamental principles involved, and then discusses the influence of the form of the jet tube, of the effective area and leakiness of the valves, and of the speed of flight on the mode of operation of the pulsejet tube and on the ratio of the amount of charge induced for the second cycle to the standard charge for the first cycle.

In Part II the sequence of pressure changes during combustion is considered. Possible means of making the operation of the jet tube more independent of flight speed and of reducing the flow losses are proposed and discussed. Diagrams and curves are presented.

Ahmed D. Kafadar, USA

872. E. V. Stoopochenko, "Influence of external friction upon the formation of shock waves in cylindrical pipes" (in Russian), *Notes Acad. Sci. USSR* (*Doklady Akad. Nauk SSSR*), Feb. 21, 1948, vol. 59, pp. 1073–1076.

Brief consideration is given to the effect of an external velocity dependent force on the behavior of a moving shock wave in a one-dimensional fluid system.

Newman A. Hall, USA

873. Maurice Giqueaux, "On the geometry of the steady-state flow of compressible fluids (Sur la géométrie des écoulements permanents des fluides compressibles)," *C. R. Acad. Sci. Paris*, Jan. 19, 1948, vol. 226, pp. 222–224.

Assume the existence of a family of surfaces orthogonal to the streamlines of a flow. Using intrinsic co-ordinates, the author expresses the vorticity vector and the continuity equation in terms of the derivatives of the flow speed and the principal curvatures of the orthogonal surfaces. Some properties are immediately inferred for the case of two-dimensional flow, such as that the equipotential lines have, in general, a point of inflection where the local speed is sonic.

S. S. Shu, USA

874. H. E. Moses, "The head-on collision of a shock wave and a rarefaction wave in one dimension," *J. appl. Phys.*, Apr. 1948, vol. 19, pp. 383–387.

This paper considers the flow of gas through a tube wherein a rarefaction wave moving in one direction encounters a shock wave moving in the opposite direction. Equations are developed from which changes in the velocity and pressure differential of the shock wave may be determined as they are influenced by the rarefaction wave. Several special cases of shock-wave magnitude are discussed.

Charles E. Crede, USA

Aerodynamics of Flight; Wind Resistance

(See also Revs. 762, 775, 827, 860, 866, 869, 885, 887, 890, 891, 893, 904, 907, 912, 917)

875. A. R. Wallace, P. F. Rossi, and E. G. Wells, "Wind-tunnel investigation of the effect of power and flaps on the static longitudinal stability characteristics of a single-engine low-wing airplane model," *Nat. adv. Comm. Aero. tech. Note*, no. 1239, Apr. 1947, pp. 1–125.

This report presents results of model tests made as part of a comprehensive investigation of the effect of power, flaps, and wing position on the static longitudinal stability of a typical single-engine low-wing monoplane. The tests were made with a full-span single-slotted flap and a full-span double-slotted flap. The horizontal tail incorporated a leading-edge slot and was placed high to avoid the slipstream. Air-flow surveys in the vicinity of the tail were made with the double-slotted flap deflected, and the wing stall was investigated by means of tufts. A considerable amount of data is presented for the various configurations tested.

David W. Whitcomb, USA

876. V. Tamburello and J. Weil, "Wind-tunnel investigation of the effect of power and flaps on the static lateral characteristics of a single-engine low-wing airplane model," *Nat. adv. Comm. Aero. tech. Note*, no. 1327, June 1947, pp. 1–76.

The tests reported are similar to those of *Nat. adv. Comm. Aero. tech. Note*, no. 1239, except that lateral characteristics were investigated. They included tests with flaps neutral, full-span single-slotted flap, and full-span double-slotted flap configurations.

David W. Whitcomb, USA

877. Stanley U. Benscoter, "Matrix development of Multhopp's equations for spanwise air load distribution," *J. aero. Sci.*, Feb. 1948, vol. 15, pp. 113–120.

The integral equation of Prandtl lifting line theory has been solved approximately by Glauert by assuming a trigonometric polynomial expansion of the nth order (n being an arbitrary integer greater than zero) for the circulation, and replacing the integral equation by a system of n equations in the n unknown coefficients of the trigonometric expansion. Solution of this system solves the problem.

In this paper the Glauert method is carried through with the use of matrix algebra. This reveals the existence of a "universal" matrix, i.e., a matrix independent of the section properties of the wing, for a given n, which plays an important role in the solution of the problem. This matrix was discovered by Multhopp [*Luftfahrtforsch.*, 1938, vol. 15, p. 153] who, however, did not employ matrix theory.

William Pell, USA

878. Clinton E. Brown and Mac C. Adams, "Damping in pitch and roll of triangular wings at supersonic speeds," *Nat. adv. Comm. Aero. tech. Note*, no. 1566, Apr. 1948, pp. 1–29.

Supersonic sources and line doublets were used to obtain the load distribution and the damping derivatives in pitch and roll at supersonic speeds for triangular wings having leading edges swept ahead of as well as behind the Mach cone from the apex. In the

latter case, analogous considerations of the surface potential distribution for low-aspect-ratio triangular wings and infinite rectangular wings were used to derive and solve the basic integral equation. Calculation of the surface potential distribution for the wings having the leading edge swept ahead of the apex Mach cone was possible by direct integration.

The analytical and graphical results presented include the stability derivatives C_{L_q}, C_{l_p} and C_{m_q}. With the leading edge ahead of the Mach cone C_{l_p} was found to be equal to one half the value calculated for an infinite rectangular wing, and C_{m_q} for pitching about the apex to be 3.375 times that of an infinite rectangular wing.

Arthur L. Jones, USA

879. **Paul E. Purser and Margaret F. Spear, "Wind-tunnel investigation of effects of unsymmetrical horizontal tail arrangements on power-on static longitudinal stability of a single-engine airplane model,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1474, Oct. 1947, pp. 1–43.

Tests were made of asymmetrical tail arrangements on a propeller-driven fighter model in an attempt to utilize the known asymmetry of the slipstream to improve longitudinal stability characteristics. The attempt appears to have been partially successful.

W. R. Sears, USA

880. **B. Maggin and C. V. Bennett, "Low-speed stability and damping-in-roll characteristics of some highly swept wings,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1286, May 1947, pp. 1–21.

This is a report of tests in a free-flight tunnel and a free-spinning tunnel of five wings, three swept back and two swept forward, varying in aspect and taper ratio. Force measurements provided lift, drag, and pitching moment data as well as the lateral sideslip stability derivatives. Damping-in-roll and tuft tests were included.

Arthur L. Jones, USA

881. **S. V. Falkovich, "On the theory of a wing of finite span in a supersonic flow"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, May-June 1947, vol. 11, pp. 391–394.

In this paper a method for the determination of the velocity potential $\varphi(x,y,z)$ of a supersonic flow past a wing of finite span is described.

Under the usual simplifying hypotheses this problem can be reduced to the determination of a solution of the equation $\varphi_{xx} + \varphi_{yy} - \varphi_{zz} = 0$, for which, on the intersection of the wing with the xz plane (the plane of symmetry of the wing) $(\partial\varphi/\partial y)_{y=0}$ is equal to a given function $\chi(x,z)$, and which vanishes on the envelope of the family of Mach cones with vertexes along the leading edge. Using the formulas of Volterra, the author obtains a representation for $\varphi(x,y,z)$ in terms of χ. The method is generalized for the case of a vibrating wing in a supersonic flow.

Stefan Bergman, USA

882. **Sydney Goldstein, "Low-drag and suction airfoils,"** *J. aero. Sci.*, Apr. 1948, vol. 15, pp. 189–220.

This paper, the eleventh Wright Brothers lecture, is notable for the first presentation of a new approximate airfoil theory which is especially suitable for designing a wing section with a specified velocity distribution and for forming quick judgments on the probable relations between the geometric and aerodynamic properties of the wing. The starting point is the exact potential-flow solution for the velocity distribution on the surface of a given airfoil section previously obtained by Theodorsen and Garrick. A first crude approximation (I) is obtained by linearizing the accurate solution. This approximation is wrong near the nose and perhaps also near the tail. A second approximation (II) consists merely of inserting a factor, obvious from the accurate solution, to remedy this. For the third approximation (III) formulas are given which come as near to the accurate solution as possible while keeping the computations simple. Although the final formula is not linear, the calculation of all the subsidiary quantities is linear. Comparisons of theoretical and experimental results are given.

For the inverse problem of designing an airfoil section with a specified type of pressure distribution on its surface, the airfoil section is designed and the approximation III of the direct problem is computed from an assumed knowledge of approximation I, the given distribution being regarded as approximation II. Applications are given to the design of low-drag airfoils. Flight test data are presented, in which the influence of surface waviness and roughness is examined.

The author emphasizes the need for using the theory in conjunction with good engineering judgment, controlled experiments, and full-scale trials.

The theory has also been applied to the design of airfoils of a special type suggested by A. A. Griffith. Griffith's idea was that if suction is to be applied to an airfoil to prevent the separation of the boundary layer, the airfoil should be designed with this in mind. If a single suction slot is to be used on each surface the desired pressure distribution is then one in which the pressure falls over the entire surface except for a discontinuous rise at the suction slot. The resulting airfoils are of unconventional shape, those illustrated being from 16 to 70 per cent thick, with the slots at about 0.8 of the chord from the leading edge. For those tested in a wind tunnel, the behavior was in accordance with theory when sufficient suction was used to prevent separation. Data are presented on the effective drag coefficients which would be realized without consideration of the losses in the slots and ducting. For a 30 per cent thick wing the ideal effective drag coefficient decreases from about 0.004 to about 0.001 as the Reynolds number increases from 10^6 to 10^8.

The author calls attention to the many unsolved problems of the suction airfoil. The practical applications are not yet clear, but it seems unlikely that such thick sections could be used at speeds greater than 400 or 500 mph.

A brief discussion is given of the application of suction to thin airfoils for increasing maximum lift, and of the idea of using distributed suction.

Hugh L. Dryden, USA

883. **Brian Thwaites, "The production of lift independently of incidence,"** *J. roy. aero. Soc.*, Feb. 1948, vol. 52, pp. 117–124.

The scheme envisaged by the author is that of an airfoil with a porous rounded trailing edge and a small flap, the setting of which can be varied. By sucking through the porous surface, separation there can be avoided, and the circulation attains the steady value corresponding to a stagnation point at the junction of the flap and airfoil. By varying the flap setting, the circulation can be varied within wide limits without altering the incidence.

It is said that such airfoils could be designed with favorable pressure distributions for extensive laminar layers over wide ranges of the lift coefficient. Further, by reducing the incidence with increase of circulation, large suctions near the nose could be avoided, thus avoiding rapid separation from the nose near the stall. It is suggested that the airfoil with flap retracted would not be responsive to gusts.

Preliminary experiments on a porous circular cylinder give promising results, separation at a lift coefficient of 5.5 (with 40-deg flap deflection) being avoided with a suction quantity given by $C_Q = 13.7/R^{1/2}$.

A. D. Young, England

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 767, 881)

884. R. E. De Haan, "Wing vibrations in still air (Standtrillingen)," *Ingenieur's-Gravenhage*, Feb. 27, 1948, vol. 60, pp. L7–12.

In this paper, which forms the subject matter of a lecture presented in October 1947, the author reviews a common method of calculating the frequencies and the modes of vibration of an airplane wing in still air. He applies the method to a model system, which was developed for reproducing the relevant properties of a wing, the model of a four-engined airplane with tanks full and empty being described.

Checks with experimental results obtained with this model are presented, with numerous figures showing the theoretical and experimental nodal lines for various modes.

J. H. Greidanus, Holland

885. Henry A. Pearson and William S. Aiken, Jr., "Charts for the determination of wing torsional stiffness required for specified rolling characteristics or aileron reversal speed," *Nat. adv. Comm. Aero. Rep.*, no. 799, 1944 (publ. 1948), pp. 1–10.

The charts presented may be used to determine the required torsional stiffness of a wing if a given loss in rolling ability is not to be exceeded. Aileron reversal speed and loss in rolling effectiveness at any other speed may also be determined. In the derivation of the charts account was taken of the effect of the span and the position of the aileron on the torsional twist distribution of the wing. The charts cover taper ratios of 0.25, 0.50, and 1.0 as well as elliptical planform, with aileron span and position as additional variables.

Edward N. Bowen, USA

886. M. A. Biot and Lee Arnold, "Low-speed flutter and its physical interpretation," *J. aero. Sci.*, Apr. 1948, vol. 15, pp. 232–236.

The significance of the nodal-line location in low-speed flutter is discussed. It is shown that the conditions for classical zero-speed flutter and for vortex free flutter are the same, namely, that the nodal line is at the three-quarter chord of the airfoil.

Conrad C. Wan, USA

887. E. Reissner and J. E. Stevens, "Effect of finite span on the air load distributions for oscillating wings—II. Methods of calculation and examples of application," *Nat. adv. Comm. Aero. tech. Note*, no. 1195, Oct. 1947, pp. 1–131.

This paper presents applications to a number of cases of the theory for the loads on an oscillating wing of finite span, developed by the senior author [*Nat. adv. Comm. Aero. tech. Note*, no. 1194, Mar. 1947]. The integral equation for the three-dimensional circulation function is solved through the use of trigonometric series in a manner analogous to that used in steady-state lifting line theory. Certain functions which are needed in the solution of this equation are tabulated. The specific problems of rectangular and elliptic wings of aspect ratio 3 and 6 are completely solved for reduced frequencies from 0.5 to 1.5, and for a number of practical deflection forms. For wings of aspect ratio 3, the three-dimensional effect is found to be important for reduced frequencies up to 1.0, and for wings of aspect ratio 6 it is important up to 0.5.

Two flutter problems are investigated employing three-dimensional aerodynamic forces. The first is a uniform rectangular wing of aspect ratio 6, which had been tested for flutter. The calculated flutter speed, employing two-dimensional aerodynamic forces, is 28.6 mph, while with three-dimensional aerodynamic forces the calculated speed is 34.2 mph at a reduced frequency of about 0.3. The measured flutter speed is about 34 mph, which is indeed remarkable agreement. The calculations are based on the assumption of a parabolic bending mode and a linear torsion mode and it is not known to what extent this may have influenced the results; in the opinion of the reviewer the error is probably not of the same order of magnitude as the three-dimensional aerodynamic effect, so that the comparison is still a good check on the theory. The second example is a case of fuselage side bending-torsion flutter involving tail surfaces of smaller aspect ratio; the three-dimensional aerodynamic theory results in a 20 per cent increase in the flutter speed.

This paper should be of great value to flutter engineers, not only because of the method of analysis presented, but also because of the considerable number of cases of air load distribution calculated. These should assist in determining the importance of span effects in many flutter analyses, and in making approximate corrections for span effects where the lengthier complete analysis is not justified.

A. H. Flax, USA

888. H. L. Runyon and J. L. Sewall, "Experimental investigation of the effects of concentrated weights on flutter characteristics of a straight cantilever wing," *Nat. adv. Comm. Aero. tech. Note*, no. 1594, June 1948, pp. 1–39.

Tests are made to determine the effects on the flutter characteristics of adding concentrated weights to a uniform straight cantilever-model wing. Several values of concentrated weight are used and the position of the weight is varied both spanwise and chordwise. A high-speed motion-picture camera is used to record the oscillations of the wing during flutter, and vibration records are obtained by means of electric strain gages and a recording oscillograph. By careful increases of the air speed of the tunnel near the flutter speed, almost 100 runs are made without damage to the model.

Tabular results of the tests are given and curves are presented to show the effect on flutter parameters of the spanwise and chordwise position of the concentrated weight in terms of the ratios of weighted to unweighted wing. In general, the flutter speed decreases and then increases as the weight is moved from root to tip. The flutter speed usually decreases as the weight is moved aft in the chordwise direction. A divergence region is determined for weights located forward of the wing center of gravity.

No correlation between test and analytical results is presented.

M. V. Barton, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 845, 860, 9.9, 922)

889. Theodore Theodorsen, "Theory of propellers," McGraw-Hill Book Co., Inc., New York, N. Y., 1948. Cloth, 6.2 × 9.2 in., 164 pp., 78 figs. and charts, $3.50.

This book presents a more general and precise treatment of propeller theory than conventional methods afford. The theory is based on conditions in the wake infinitely far from the propeller; several new parameters refer to the far-distant wake and not to the propeller. The theory is generally based on heavily loaded propellers and is not subject to the usual limitations of light loadings. Single and dual propeller design and selection problems are logically studied, the methods being fortunately very similar to earlier methods.

The chapter contents are: I. Discussion of general problem.

II. Conditions for optimum distribution of circulation along the blade. III. Solution to the problem of optimum distribution of heavily loaded, single-rotation propellers. The differential equation for the dual-rotation propeller is also given. IV. Most of the new expressions for ideal thrust, torque, and efficiency, as well as the loss factors, all based on wake parameters, are introduced. All the important relations are obtainable from a so-called mass coefficient which physically represents the "effective" cross section of the propeller in terms of the propeller disk area. V. An experimental electrical method for obtaining the loading function and the mass coefficient is given. VI. Application of the theory to propeller design problems, including brief discussions of special effects such as compressibility, cavitation, "dead water," and others. The general procedure is given briefly and a four-bladed heavily loaded propeller is used as an example. VII. The problem of slipstream contraction is covered, and an empirical relation for technical use is given. VIII. Propeller selection problems, including conventional parameters and contour charts. IX. Problem of body interference on the propeller.

The book contains many useful tables and graphs throughout, including graphs of the circulation function and mass coefficient, for single and dual propellers with 2, 4, and 2, 4, and 6 blades respectively. H. P. Liepman, USA

890. Theodore Theodorsen, "The theory of propellers—Parts 1, 2, 3, 4," *Nat. adv. Comm. Aero. Rep.*, nos. 775, 776, 777, and 778, 1944 (issued in 1948), pp. 1–17, 1–21, 1–23, 1–7.

This series of reports presents a new theory of propellers based entirely on conditions in the far-distant wake of the propeller. Thrust, torque, and efficiency are all uniquely given as functions of the ultimate wake only. The theory is applicable to propellers of heavy and light loadings and is developed for single- and dual-rotation propellers including stationary guide vanes.

Part 1, "Determination of the circulation function and the mass coefficient for dual-rotating propellers," gives the circulation functions for 4, 8, and 12-blade dual-rotating propellers for advance ratios from 2 to about 6 and also for some single-rotating propellers. The mass coefficient is introduced as the most important parameter of this theory, and is given for the same dual-rotating propellers and some single-rotating propellers. Values of the mass coefficient are obtained by integration of the circulation function or by special measurements on an electrical device.

Part 2, "Method for calculating the axial interference velocity," gives a technical method with numerical values for the calculation of this velocity for 2, 3, and 6-bladed propellers.

Part 3, "The slipstream contraction with numerical values for 2-blade and 4-blade propellers," presents the relation between the propeller diameter and the wake diameter.

Part 4, "Thrust, energy, and efficiency formulas for single- and dual-rotating propellers with ideal circulation distribution," gives the expressions, tables, and graphs for practical use of the theory presented in this series. H. P. Liepman, USA

891. M. J. Zucrow, "Principles of jet propulsion and gas turbines," John Wiley and Sons, New York, N. Y., 1948. Cloth, 6 × 9.3 in., 563 pp., 235 figs., $6.50.

As the reference to "jet propulsion" in the title implies, this book is oriented toward applications in aircraft propulsion. The emphasis is principally on thermodynamic and aerodynamic aspects, with one chapter on high-temperature metallurgy prepared with the help of C. T. Evans, Jr. The first half is devoted to a review of fluid properties, momentum and energy relations, thermodynamics of gas flow, airplane performance, and propeller analysis. The one-dimensional treatment of flow in ducts is covered quite thoroughly. The second half takes up gas-turbine-cycle studies, turbojet-cycle analyses, centrifugal and axial air compressors, turbine characteristics and types, combustion chambers, rocket motors, and high-temperature metallurgy. Ramjets and pulsejets are mentioned in passing.

The cycle analyses are carried out assuming perfect gas, negligible increase in mass flow through the combustor, and, except for a few discussions, negligible pressure drops in piping, regenerator, and combustor. Static temperatures and pressures appear to be used in these studies.

The chapters on centrifugal and axial compressors present briefly their engineering fundamentals. The chapter on turbines is relatively more complete. The brevity of the treatment of the combustion chamber is no doubt due to the classified status of much of the material. Pressure loss is analyzed in terms of static rather than total pressure drop. Stability of burning is given passing mention as an important problem.

Relatively complete treatment is given of the rocket motor, except for details of thermochemical calculations. Jet-velocity calculations, thrust, and performance analyses are given, along with a discussion of fuel combinations. Burning-chamber and nozzle construction details, still highly classified, could be discussed only in very general fashion. Stewart Way, USA

892. J. R. Weske, "Fluid dynamic aspects of axial-flow compressors and turbines," *J. aero. Sci.*, Nov. 1947, vol. 14, pp. 651–656.

The ideal operation of radial elements of axial turbomachine stages, designed for flow on coaxial stream surfaces and having identical inlet and outlet velocity magnitudes and directions, is shown to be determined by three dimensionless parameters: (1) The flow coefficient, or ratio of axial-flow velocity to wheel speed; (2) the pressure coefficient, or ratio of stage total head change to wheel speed velocity energy; and (3) the reaction coefficient, or ratio of rotor static head change to stage total head change.

A chart is presented giving the velocity diagrams and schematic blade shapes for such stage elements, with a unity flow coefficient and various pressure and reaction coefficients covering compressor and turbine operation. The radial variation of pressure and reaction coefficients and schematic blade shapes is also shown for two turbine stages designed for a constant axial velocity along the radius, one having the energy input constant along the radius, and the other having the reaction coefficient constant along the radius.

A qualitative discussion is presented of the three-dimensional aspects of the flow resulting from the action of fluid viscosity, based upon the flow in a curved channel, the action of surface boundary layers and blade wakes, and the centrifugal action in rotors. Diagrams are given for turbine stages which show qualitatively the boundary layers and blade wakes for stationary and moving rows, with and without axial exit flow.

A criterion for stability of flow on coaxial stream surfaces is proposed, based upon the balance of centrifugal forces and radial pressure gradients. W. G. Cornell, USA

893. Henry Parkus, "The disturbed flapping motion of helicopter rotor blades," *J. aero. Sci.*, Feb. 1948, vol. 15, pp. 103–106.

This article is a theoretical treatment of the stability of a helicopter rotor blade. The rotor blade is assumed to have a flapping hinge on the axis of rotation. The blade mass coefficient, coupling between blade angle and flapping, and the flight to tip speed ratio are considered. The analysis shows that the disturbed blade executes nonperiodic oscillations which are

described by a linear differential equation with periodic coefficients. A condition of stability is given which permits the determination of the stability without solving the general equation.

It is shown that for practical values of the variables, the blade is highly damped and that the disturbance is nearly extinguished in one revolution of the rotor. Arthur A. Regier, USA

894. S. Katzoff, Harriet E. Bogdonoff, and Howard Boyet, "Comparisons of theoretical and experimental lift and pressure distributions on airfoils in cascade," *Nat. adv. Comm. Aero. tech. Note*, no. 1376, July 1947, pp. 1–19.

Experimental pressure-distribution data for cascades simulating axial-compressor blades and turbine blades are compared with theoretical incompressible-flow calculations made by the interference method [Katzoff, Finn, and Laurence, *Nat. adv. Comm. Aero. tech. Note*, no. 1252, May 1947]. For a given angle of attack the experimental lift coefficients were considerably less than the theoretical values. When the pressure distributions were compared at a given lift coefficient, however, fair agreement between theory and experiment resulted. It is pointed out that a part of the discrepancy between the theoretical and experimental lifts at a given angle of attack may be due to the effects of side-wall boundary layers. In order to evaluate this effect the authors suggest that cascade tests of high-aspect-ratio blades should be made. John V. Becker, USA

Experimental Flow Equipment and Technique

(See also Revs. 888, 925)

895. C. H. Lindsley and E. K. Fischer, "End effect in rotational viscometers," *J. appl. Phys.*, Nov. 1947, vol. 18, pp. 988–996.

Numerous tests are reported with a commercial Stormer viscometer the geometrical characteristics of which were widely varied. The tests were to determine the so-called "end effect" for cups and bobs of several sizes. The authors first evaluated various methods for eliminating or measuring the end effect, and decided for the method of multiple bobs, that is, comparing the ratio of driving torque to rpm for a number of bobs of constant diameter but varying length.

The results of the tests give an interesting picture of the influence of characteristics of geometrical and dynamical similarity which might be used to establish a more general theory of the rotational viscometer. A rather striking conclusion is that air trapped in the hollow space of an open bob (without bottom) does not essentially change the end effect.

The practical results which can be applied to any rotational viscometer are: With the method of multiple bobs an accuracy of ±2 per cent can be obtained without calibration. When the instrument is calibrated by the use of standard liquids or when its constant is computed from a value for the end effect previously found by methods such as the authors applied, the accuracy is lower but should be within ±5 per cent.

Wilhelm Spannhake, USA

896. Edward W. Otto, "Analysis of accuracy of gas-filled bellows for sensing gas density," *Nat. adv. Comm. Aero. tech. Note*, no. 1538, Feb. 1948, pp. 1–29.

An analysis is presented to determine the cause for error of gas-filled bellows. Some interesting results are derived, showing that low values of filling volume and large diaphragm areas with low spring rates are desirable. Methods of redesign for the smallest error are shown, and the effects of each bellows design constant are indicated by graphs. H. E. Sheets, USA

897. R. Betchov, "The thermic inertia of hot-wire anemometers and the approximate calculation of their characteristics (L'inertie thermique des anémomètres à fil chaud et le calcul approché de leurs caractéristiques)," *Proc. kon. Ned. Akad. Wet.*, Feb. 1948, vol. 51, pp. 224–233.

This article describes a method of determining the thermal inertia of a hot-wire anemometer by imposing on the wire an oscillating electric current over a wide range of frequency.

In a previous paper, the author has developed differential equations interrelating the change of resistance and temperature of a hot wire when subjected to an oscillating electric current. The magnitude of the superimposed oscillations is assumed small in comparison to the heating current, so that second-order terms may be neglected. The thermal inertia of wires of different diameters has been determined by calibration in an alternating-current bridge over a range of frequencies. A description is given of the calibration circuits used, and the subjects of corrections, errors, compensation, etc., are discussed in detail.

Frank L. Wattendorf, USA

898. C. B. Millikan, J. E. Smith, and R. W. Bell, "High-speed testing in the Southern California Cooperative Wind Tunnel," *J. aero. Sci.*, Feb. 1948, vol. 15, pp. 69–88.

The article gives a discussion of a rather large number of problems of testing in this variable density, high subsonic speed, 8.5 × 12-ft wind tunnel, and the results of some recent (calibration) tests.

Some of the aspects discussed are: (*a*) Flow characteristics, such as Mach number M and pressure distributions, wall boundary-layer profiles, turbulence measurements with hot wires (the turbulence at atmospheric pressure is much greater than when the tunnel pressure is one-half atmospheric pressure) and with spheres (this method cannot be used at large M). (*b*) Remarks about high-speed operating procedure; the choking Mach number is given as a function of blocked area. (*c*) Methods of model support are discussed; experimental curves of drag tares and pitching moment tares as a function of α and M are given for vertical and swept struts. (*d*) The effects of compressibility on blocking and other tunnel-wall corrections are discussed in particular detail (with reference to literature).

Besides the high subsonic range, the method of testing small models in the transonic region above a thick-cambered airfoil-like "Lockheed Bump" is also discussed. E. Dobbinga, Holland

899. Lars Malmquist, "Temperature measurements in high-velocity gas streams" (in English), *Trans. roy. Inst. Technol. Stockh.*, 1948, no. 15, pp. 1–52.

The paper concerns a method of measuring the stagnation temperature of gas streams with high velocity, and starts with a discussion of the theoretical basis of the method. In a similar investigation by Hottel and Kalitansky [*J. appl. Mech.*, Mar. 1945, vol. 12, p. 24] the temperature probe consisted of a single tube open at the front with a thermocouple junction fitted centrally in the tube downstream of vent holes. The recovery factor for the dynamic temperature varied from 90 to 98 per cent at velocities from 10 to 100 per cent of sound velocity.

In the present investigation, after starting with tests on similar probes, tests were made on improved types of this probe. By placing the vent holes downstream of the thermocouple junction,

to give a more uniform and effective heat transfer to the junction, the recovery factor was increased to above 96 per cent through the whole velocity range. After providing a double tube, to give an insulating layer of air between the tubes, and further improvements, the recovery factor with the optimum vent-hole area was around 99.8 per cent through the whole velocity range. Calculations indicate that test results are reliable and that the reproducibility of the probe is good. Complete drawings of the probe are presented.

Tests were made with both cold and hot air (about 250 C) discharged into free air. The error of measurement was less than 0.3 C at subsonic velocities even if the difference between the stagnation temperature of the gas and the temperature of the radiant surfaces in the surroundings was as high as 300 C. The method can therefore be used for accurate temperature measurements in free gas streams at relatively high temperatures.

Ahmed D. Kafadar, USA

900. W. P. Mason, "Viscosity and shear elasticity measurements of liquids by means of shear vibrating crystals," *J. Colloid Sci.*, May 1948, vol. 3, pp. 147–162.

When a vibratory source of shear stress is immersed in a liquid, it excites viscous shear waves so highly attenuated that sometimes thin films of liquid suffice for test. The equivalent electric circuit for the reaction of the liquid is a transmission line with constant series mass and shunt resistance per unit length. When the wave source is a crystal, this loading results in an increased electric resistance and a lowered resonant frequency as compared to the behavior in vacuo, and the equivalent parameters may be calculated.

By experiment it was found that for many low-viscosity liquids the equivalent shunt resistance was due to a constant viscosity, essentially that obtained by conventional methods. However, for high-viscosity liquids the behavior at high frequencies requires that to the above equivalent transmission line there be added a shunt elasticity element arising from shear stiffness not considered in the simple theory. It is related to the asymptotic high-frequency electric resistance. While some of the high-viscosity liquids tested followed the behavior of this three-element line, some showed evidence of the presence of two shear-elastic elements, which may be interpreted in terms of the geometry of the polymer molecules investigated. Vincent Salmon, USA

Hydraulics; Transport of Solids; Cavitation

(See also Rev. 858)

901. H. Proetel, "Model tests and studies for the design of locks utilizing the kinetic energy of the water (Modellversuche und Planungen für die Ausbildung der Schwungschleusen)," *Bautechnik*, Jan. 1948, vol. 25, pp. 3–15.

In the locking operation in a canal a considerable reduction in water loss is obtained by discharging not directly to the lower level but to catch basins beside the lock. The author describes a special arrangement of lock and catch basins whereby the water loss can be reduced even more by utilizing the kinetic energy of the water flowing in or out of the lock to produce a surface differential between lock and basin, and holding this differential by properly timed opening and closing of the discharge valves.

The working and efficiency of these "pendulum locks" were investigated by model tests which were begun in 1932 and are still continuing. Numerous variations of design were investigated. The results of further tests will be given in future papers.

Typical of the results obtained are those on a 1 to 24 model of a proposed lock at Munster. Without the use of catch basins the lock chamber could be filled in 10.8 min full scale and emptied in 9.5 min, without exceeding the allowable forces on the mooring ropes of the tow which were fixed at $^1/_{600}$ of the tow weight. With the use of ordinary catch basins, the times of filling and emptying were about the same, but the water loss was reduced by 50 per cent. With the pendulum lock arrangement the filling time was 14.9 min and the emptying time 12.6 min but the water loss was reduced by 62.7 per cent. In other cases the water loss was reduced as much as 75 per cent. Karl E. Schoenherr, USA

902. H. C. Brinkman, "A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles," *Appl. sci. Res. Sec. A*, 1947, vol. 1, no. 1, pp. 27–34.

By solving a form of Darcy's equation, arbitrarily modified to permit extension to low porosities, the author obtains an expression for the viscous force on a group of closely spaced particles. A single spherical particle within the mass is considered for the integration and the definition of appropriate boundary conditions.

The drag per unit volume is expressed in terms of the permeability, the result comparing favorably with an empirical result by Carman based on data of limited range. Restrictions on the applicability of the results for very small percentages of void space are indicated. John S. McNown, USA

Marine Propulsion

(See also Revs. 762, 768, 918)

903. Sir Amos Ayre, "Approximating ehp; revision of data given in papers of 1927 and 1933," *Trans. N. E. Coast Instn. Engrs. Shipb.*, Mar. 1948, vol. 64, pp. 211–222.

This revises an earlier empirical formula giving the effective horsepower (ehp) required by a ship of stated dimensions and speed. It rests upon model tests, but does not require these in specific applications. The ehp is computed from displacement, speed, and a "constant" C_2 which is a function of the speed-length ratio for a standard ship. The article describes methods of adjusting C_2 for variations from the standard in block coefficient, beam-draft ratio, location of the center of buoyancy, and shape of stern. A revised chart gives values of C_2 for speed-length ratios from 0.3 to 1.9 and length-displacement ratios from 10 to 25. Tables and formulas for C_2 and an illustrative computation are also presented. The original articles must be read for complete information. A. O. Williams, Jr., USA

904. L. Sedov, "Scale effect and optimum relations for sea surface planing," *Nat. adv. Comm. Aero. tech. Memo.*, no. 1097, Feb. 1947, pp. 1–36 [transl. from *Cent. Aero-Hydrodyn. Inst.* (*Tsent. Aero-Gidrodin. Inst.*) *Rep.*, no. 439, 1939, Moscow].

The article begins by a discussion of the use of dimensional analysis for studying the lift force and the resistance of a planing surface, and develops the customary Reynolds number, Froude number, lift and drag coefficients, and load as displacement coefficient. A semiempirical equation previously developed by the author for the lift of a planing flat plate is presented and compared with experimental data and other empirical formulas.

At large Froude numbers the equation and experiments indicate that the lift coefficient is relatively independent of the Froude number. It is next shown that the resistance is also

independent of the Froude number, and it is developed on the basis of the Prandtl formula for resistance of a smooth flat plate oriented parallel to the towing velocity.

An analysis of the scale effect (differences in the ratio of resistance to lift, between the model and its prototype) is made for experimental data obtained at the optimum planing condition, and also for data obtained by improperly running tests at a constant Froude number. Using the equations for lift and resistance which he has developed, the author is able to determine mathematically the values of optimum planing angle and optimum width of a planing flat plate.

Naval architects have developed reliable means of determining the resistance of hulls at low Froude numbers, which satisfy the requirements for low- and moderate-speed ships. The author has contributed a useful analysis of resistance at high Froude numbers, but there is still a wide range yet to be covered. F. Everett Reed, USA

Lubrication, Wear

905. S. A. McKee, H. S. White, and J. F. Swindells, "Measurements of combined frictional and thermal behavior in journal-bearing lubrication," *Trans. Amer. Soc. mech. Engrs.*, May 1948, vol. 70, pp. 409–418.

Data were obtained in tests with a four-bearing friction machine which show that an increase in the load on a journal bearing produces a proportional increase in frictional torque when other conditions of the test are held constant. Under these same conditions an increase in load also produces a proportional increase in the fluidity of the oil in the bearing. These two effects are the result of the combined hydrodynamic and thermodynamic actions involved in journal-bearing operation with forced-feed lubrication.

The increase in torque is influenced by the viscosity of the oil, the oil-inlet temperature, the oil-feed pressure, the shaft diameter, the clearance-diameter ratio, and the length-diameter ratio. The increase in fluidity is influenced by the same factors and also by the speed of the journal. Empirical equations are derived for the fluidity-pressure relationship for certain conditions. Also, a graphical method is given for the use of this relationship in estimating safe bearing loads.

R. C. Binder, USA

906. R. L. Johnson, Douglas Godfrey, and E. E. Bisson, "Friction of solid films on steel at high sliding velocities," *Nat. adv. Comm. Aero. tech. Note*, no. 1578, Apr. 1948, pp. 1–65.

This research is a continuation of a study reported by Johnson, Swikert, and Bisson [*Nat. adv. Comm. Aero. tech. Note*, no. 1442, Oct. 1947]. The authors report the results of friction and wear studies made with surfaces coated with various inorganic films which are of importance in run-in, in extreme-pressure lubrication, and in supplementary or dry lubrication. Additional studies were made using standard physical, chemical, and metallurgical techniques, including electron diffraction. The relation of the findings to friction and lubrication theory and practice is discussed.

The apparatus used for the friction and wear tests consisted basically of an elastically restrained spherical steel rider sliding on a rotating steel disk. The range of sliding speeds employed was from 50 to 8000 fpm with loads from 169 to 1543 grams (108,000 to 225,000 psi, initial Hertz surface stress). The inorganic films studied were α-Fe_2O_3, α-Fe_3O_4, $FeCl_2$, FeS, MoS_2, and graphite. These were applied to the disk surface only. Of these, Fe_3O_4, $FeCl_2$, MoS_2, and graphite were found to be the most beneficial as regards friction and wear. M. E. Merchant, USA

Dynamics of Meteorology and Oceanography

(See also Revs. 773, 781, 915)

907. I. I. Gringorten and H. Press, "A meteorological measure of maximum gust velocities in clouds," *Nat. adv. Comm. Aero. tech. Note*, no. 1569, Apr. 1948, pp. 1–20.

Each of the many aspects of atmospheric turbulence leads to complicated problems, and results lending themselves to forecasting for aviation have not been many. Recently, statistical approaches to some of these problems have appeared hopeful. The present study of gusts in clouds is among the few to give directly usable results.

The authors are able to correlate significantly the maximum effective gust velocity within a convective cloud, to a function of two meteorological parameters. These parameters are the height of the convective activity and the spread in temperature along a horizontal traverse, used because of their relation to the amount of kinetic energy available to the cloud particles.

It is likely that the success of this attack is due in large part to the decision to relate maximum gustiness to the macroscopic characteristics of the cloud, in contrast with less successful previous endeavors which attempted to connect the gustiness found at given levels with the values of local meteorological variables.

Joanne G. Starr, USA

908. Johan Van Veen, "Analogy between tides and alternating currents (Analogie entre marées et courants alternatifs)," *Houille blanche*, Sept.-Oct. 1947, vol. 2, pp. 401–416.

Most geophysical problems appear almost hopelessly complex from a dynamical point of view, due largely to the great number of forces acting and to complicated boundary conditions. Often a direct frontal attack leads to equations not feasible to solve, or if solvable equations are deduced a price is paid in such drastic approximations that the results bear little resemblance to observations. Hence analogies from other physical sciences should not be ignored by the geophysicist, since they often provide both a clearer physical model and a reduction of the complexities to a level where computations can be made.

Tidal phenomena have been greatly beset by complexity, and therefore the analogy presented by the author between tides in canals and alternating electric currents is a most fortunate one. It provides both a departure point for understanding tidal phenomena in bounded regions, and lends itself to calculations by engineers who must predict currents, water heights, etc., in numerous channels without unduly laborious mathematics.

In dealing with tidal phenomena within boundaries, a far more complex problem is presented than is the case with long waves on the open ocean. In fact this problem gives rise to a complicated form of the telegraphy equation, only linear under certain restrictions. Thus, in order to make the electrical analogy a significant one, it is extended to an examination of the "circuit" parameters in the canals. The crucial correspondence is that between electromotive force and the slope of the water surface.

Once the physical analogy is so clarified, it is possible to adopt far simpler electromagnetic equations, such as Ohm's and Kirchoff's laws. This is done with a high degree of predictability for canals containing many branches, plus elements corresponding to inductance (inertia) and capacitance (storing). By thus making a comparison with directly observed hydraulic quantities,

both computation and theory are benefited. It is only to be regretted that the author has not discussed a specific example of utilization of actual observations. Joanne G. Starr, USA

909. O. G. Sutton, "The problem of diffusion in the lower atmosphere," *Quart. J. roy. met. Soc.*, July-Oct. 1947, vol. 73, pp. 257–281.

In this paper the forecasting of the diffusion of a gas or of smoke which is being emitted from a continuous line or from point sources is described. The theory applies only to smooth plains and for an approximately adiabatic lapse rate.

The report is of interest and value since it is an integrated study of atmospheric diffusion and indicates excellent agreement between experiment and theory.

The report covers work done at Porton, England, between 1932 and 1938, some of which has been previously reported in fuller detail. Philip Donely, USA

910. Victor P. Starr, "Momentum and energy integrals for gravity waves of finite height," *J. mar. Res.*, 1947, vol. 6, no. 3, pp. 175–193.

It is most gratifying that the present revival of interest in the problem of water waves includes also studies patterned along the classical investigations of the nineteenth century. The author deals with plane, irrotational waves, in water of constant (but not necessarily large) depth, which move without change in form. The investigation is not restricted to waves of small amplitude.

The starting point of the investigation is the derivation of relationships between two sets of streamline functions and velocity potentials. One set of these functions pertains to the progressive wave motion, the other set to the steady motion resulting from the superposition upon these progressive waves of a uniform flow moving opposite to the waves at wave velocity. The approach leads to five very general and *exact* relationships involving the kinetic and potential energies, horizontal momentum, wave velocity, and mass of periodic and solitary waves.

Most of the known solutions for waves of finite height are in the form of infinite series, and these series often converge too slowly to permit numerical calculations. With few exceptions, past investigations dealt only with deep water. It is of considerable interest, therefore, that the exact relations derived by the author exist, that they are relatively simple, and that they apply also to the much more complicated case of shallow water.

Walter H. Munk, USA

911. Victor P. Starr, "A momentum integral for surface waves in deep water," *J. mar. Res.*, June 30, 1947, vol. 6, pp. 126–135.

The work of Levi-Civita, on the relationship between the momentum and kinetic energy of irrotational and periodic surface waves of finite height, has been extended by the author to cover some applications in oceanography. By using Green's theorem, it is shown that the kinetic energy per wave length and per unit distance along the crests of waves is equal to one half the wave speed multiplied by the momentum of the same water mass in the direction of wave propagation. For the case of an initially undisturbed ocean of large dimensions, subjected to a constant and uniform wind, the wave length is calculated to increase as the two-thirds power of time.

The extent to which the theoretical results obtained by the author are directly applicable to ocean surface waves is an open question, as natural waves are irregular and also the medium in which they exist departs considerably from an ideal fluid. Moreover, Coriolis forces, not considered by the author, play an important part in the momentum of waves.

The relationship obtained can be further expanded to study the wave growth and drift currents in oceans due to wind action.

S. K. Ghaswala, India

Ballistics; Detonics (Explosions)

(**See also Revs. 774, 918**)

912. J. B. Rosser, R. R. Newton, and G. L. Gross, "Mathematical theory of rocket flight," McGraw-Hill Book Co., Inc., New York, N. Y., 1947. Cloth, 6.2 × 9.2 in., 276 pp., 41 figs., $4.50.

This is an official report of work done at the Allegany Ballistics Laboratory during the war, on external ballistics of rocket projectiles. It concentrates mainly upon the flight dynamics of small fin-stabilized projectiles, without wings, launched with high accelerations from aircraft or from the ground, and having a time of flight of only a few seconds. The main object is the calculation of dispersion resulting from misalignment of rocket thrust or inaccuracies in manufacture.

An opening chapter contains a rigorous derivation of the equations of motion of such projectiles. The aerodynamic derivatives required are not treated theoretically, but typical numerical values are given. Approximate solutions of the equations of motion are then used to calculate the dispersions produced by the launching process, during free flight with the rocket motor burning, and for the remainder of the flight after burning. The motion during burning is treated very fully. The authors have simplified the solution by the introduction of "rocket functions," allied to Fresnel integrals, having the form $ie^{iw}\int_w^\infty e^{-ix}dx/\sqrt{x}$. Properties of these functions are given in considerable detail. R. Smelt, England

913. R. Sänger, "Conversion of the altitude of a gun's location to the range-table altitude (Reduzierbarkeit der Geschützhöhe auf die Bezugshöhe der Flugbahnkarte)," *Schweiz. Arch.*, Mar. 1948, vol. 14, pp. 72–76.

Swiss topography would make necessary at least three range tables for each type of gun if it were not possible to convert the altitude of a gun's location to the altitude given in the available tables. The effect of altitude considered here is that of decreasing air density, which increases the range. The deviation of air density from the standard is considered through a "ballistic air density," calculated by the Dufrénois method. The author presents this method and discusses the limits of its application.

Ballistic air density and the correction for it are also discussed by Bliss ["Mathematics for exterior ballistics," John Wiley and Sons, 1944]. It is interesting to compare the two methods, which seem to attack the problem from very different points of view. W. C. Johnson, Jr., USA

914. Eugène Leimanis, "Integration by quadratures of the flight of a projectile in a medium of variable density and temperature (Sur l'intégration par quadratures des équations du mouvement d'un projectile dans un milieu de densité et température variables)," *C. R. Acad. Sci. Paris*, 1947, vol. 224, June 9, pp. 1618–1620; June 23, pp. 1752–1754.

In these papers the author gives the differential equations of exterior ballistics for the case where the air density decays

exponentially with altitude. In addition the air temperature is assumed constant in the first paper and to depend linearly on the altitude in the second paper. The author obtains solutions of the system of equations by quadrature, under the very special assumption that this solution is independent of the ballistic coefficient. The methods used are formal throughout.

Benjamin Epstein, USA

915. E. Roth and R. Sänger, "The effect of the wind on the normal trajectory (Der Wind als Störung der Normalflugbahn)," *Schweiz. Arch.*, Apr. 1948, vol. 14, pp. 108–118.

The authors consider the influence of the wind on the calculated trajectory (calculated in the absence of winds) of a projectile. A discussion of the kinematics of winds is given, with special consideration devoted to the so-called "homogeneous" wind (velocity vector invariant in space and in time). Rather less space is devoted to velocity variations with altitude. Examples of the calculations for the effects of horizontal winds are given.

A discussion of projectile drag at both subsonic and supersonic velocities is also given, partially to support some of the assumptions made in the kinematical analysis.

John W. Miles, USA

916. J. G. Kirkwood and R. J. Seeger, "Surface waves from an underwater explosion," *J. appl. Phys.*, Apr. 1948, vol. 19, pp. 346–360.

A general expression has been derived theoretically for the pressure due to surface waves produced by an underwater explosion. The theoretical treatment is an application of the standard procedure for surface waves [Lamb's "Hydrodynamics," chap. 9]. The effect of the shock wave is assumed to be negligible, so that the only source is the pulsating gas globe, assumed to be spherical. The flow is further assumed to be irrotational. The equation of continuity and Laplace's equation for the velocity potential are solved, subject to the boundary conditions of zero normal component of velocity at the fixed bottom and atmospheric pressure at the free surface.

In the application of the theory, it is assumed for computational simplicity that the volume of the gas globe, a function of the time, can be replaced by some average volume during the first pulsation. The integrals involved in the theory have been evaluated in terms of dimensionless variables over a range of practical interest. Employing the theory of D. C. Campbell for the pulsations of the gas globe in an underwater explosion [*David Taylor Model Basin Rep.*, no. 512], a specific formula is given for the pressure on the bottom of the water due to the first pulsation of the gas globe from a TNT explosion originating also on the bottom.

Stuart R. Brinkley, Jr., USA

917. Bernardino Lattanzi, "Aerodynamic tests on torpedo models with aerial tail units (Esperienze aerodinamiche su modelli di siluri muniti di aeroimpennaggi)," *Monogr. sci. Aero. Suppl. Tecn.*, May 1947, no. 3, pp. 1–21.

Since aerial torpedoes require larger stabilizing surfaces for their air trajectory than for the underwater flight, it is common to design aerial appendages which are discarded by the torpedo upon impact with the water surface. The author reports the results of model tests in a wind tunnel on a series of 14 torpedo forms equipped with various types of aerial tail surfaces. Graphs are presented of drag coefficient against Reynolds number, and graphs of lift, drag, and pitching and rolling moment coefficients against angle of pitch.

Tests were also conducted to determine the effect on the drag of a small flat plate, normal to the stream, attached ahead of the nose of the torpedo by means of a longitudinal spindle. The purpose of this plate, which is also to be discarded upon impact with the water surface, is to reduce the aerial speed of the torpedo. Graphs are given showing the effect of various sizes of plate on the drag coefficient.

Louis Landweber, USA

Thermodynamics

(See also Revs. 871, 891, 896, 899, 926, 927)

918. René Simard, "Hydrogen peroxide as a source of power," *Engng. J. Montreal*, Apr. 1948, vol. 31, pp. 219–225.

Concentrated hydrogen peroxide was developed by the firm of Walter, Kiel, as a source of high power for short periods. The author describes briefly the uses which the Germans made of it during the war years. Applications include submarine propulsion, rocket motors, and the flying-bomb launcher. Other proposed uses of hydrogen peroxide in war and peace are listed; in particular, details are given of its application to ramjets and to the boosting of conventional aero engines.

R. Smelt, England

919. A. L. London, "Gas-turbine plant combustion-chamber efficiency," *Trans. Amer. Soc. mech. Engrs.*, May 1948, vol. 70, pp. 317–328.

To derive an expression for the efficiency of a gas-turbine combustion chamber, three distinct concepts are employed, as follows: (1) A combustion efficiency which accounts for incomplete chemical conversion of the fuel and for heat losses; (2) a mechanical efficiency which accounts for flow friction losses and for auxiliary-drive mechanical energy requirements; and (3) an over-all efficiency which expresses the combined effect of these losses on the gas-turbine plant efficiency. Proposed standard methods are presented for the evaluation of these efficiency concepts, consideration being given to thermodynamic exactness and ease of application.

While there is little argument with the need for the three basic concepts, there is some question as to the methods of evaluation. In the reviewer's opinion the proposed expression for evaluation of the combustion efficiency is not in accordance with the usual definition of combustion efficiency. Also, the proposed expression for evaluation of the mechanical efficiency does not seem to specifically acknowledge pressure losses due to change in momentum of the gases passing through the combustion chamber.

J. Howard Childs, USA

920. J. L. Finck, "Thermodynamics, Part I: The second law from the standpoint of the equation of state," *J. Franklin Inst.*, Apr. 1948, vol. 245, pp. 301–317.

The concepts "equilibrium" and "reversibility" are examined with regard to the number of independent variables required to define a thermodynamic system. The author shows that the number of independent variables selected is crucial to the basic concepts as well as to the second law of thermodynamics, and that if this number is less than the minimum required to completely define the system, the equation of state for the internal energy will be incomplete. It is common practice to employ "incomplete" equations of state, and this results in processes which we consider to be irreversible.

In the author's views, both the Kelvin-Planck and Clausius principles can be considered to be statements of the fact that we always use incomplete equations of state, and therefore find

certain energy-conversion processes which would be permitted by the first law of thermodynamics to be impossible. If these equations were complete, that is, if the number of independent variables were not less than the minimum required to completely describe the system, we should have complete conversion of heat into work, as well as work into heat.

The entropy concept is considered in detail and it is shown that there are limitations in its application if complete equations of state are utilized. The difficulties in realizing complete equations of state are discussed. J. Howard Childs, USA

921. **NACA Subcommittee on Combustion, "Symbols for combustion research,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1507, June 1948, pp. 1–5.

A panel of the NACA Subcommittee on Combustion has prepared a list of standard combustion symbols, their dimensions and typical units, to facilitate the exchange of technical information and to provide a basis for comparison of results. In so far as possible, the list is consistent with accepted usage in the fields of thermodynamics and aerodynamics.

John E. Goldberg, USA

922. **Peter Lloyd, "Determination of gas-turbine combustion-chamber efficiency by chemical means,"** *Trans. Amer. Soc. mech. Engrs.*, May 1948, vol. 70, pp. 335–341.

Standard methods of gas analysis are discussed and compared with reference to their usefulness for the analysis of the exhaust stream of gas-turbine combustion systems. The N.G.T.E. Gravimetric Method is considered the best for this purpose. Some typical results of combustion losses deduced from such chemical analysis are compared. Serge Gratch, USA

923. **I. Prigogine and R. Defay, "Thermodynamic method of Th. De Donder and of Schottky, Ulich, and Wagner (Méthode thermodynamique de Th. De Donder et méthode thermodynamique de Schottky, Ulich, et Wagner),"** *Bull. Acad. Belg. Cl. Sci.*, 1947, vol. 33, no. 5, pp. 222–232.

De Donder, as well as Schottky, Ulich, and Wagner, has attempted to develop a consistent thermodynamic treatment of chemical reactions. The authors try to show that De Donder's method is preferable since the other method leads to ambiguities in the case of nonisothermal reactions. Serge Gratch, USA

924. **I. Prigogine and R. Defay, "The stability of azeotropic transformations (La stabilité des transformations azéotropiques),"** *Bull. Acad. Belg. Cl. Sci.*, 1947, vol. 33, no. 1-2, pp. 48–63.

In this paper it is shown that transfer of matter between the vapor and liquid phases of a binary solution can be stable only for azeotropic transformations.

For positive azeotropic systems the transformation is stable on condensation, unstable on evaporation. For negative azeotropic systems the transformation is stable on evaporation, and unstable on condensation. Serge Gratch, USA

925. **Albert E. von Doenhoff, "Note on similarity conditions for flows with heat transfer,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1577, Apr. 1948, pp. 1–14.

This paper presents a generalized discussion based on dimensional analysis of the problem of obtaining similarity conditions in gas flows with heat transfer. The importance of the various flow parameters such as the Reynolds, Mach, Froude, Prandtl, and Nusselt numbers is indicated. Relations involving the ratio of the heat conductivity to the viscosity and the ratio of specific heats are also indicated.

Applications to the problem of maintaining similarity conditions in wind-tunnel testing at high Mach numbers are discussed, it being suggested that considerable simplification of the problem is obtained by separating the effects of heat transfer by conduction and by radiation. M. J. Thompson, USA

Heat Transfer

(See also Revs. 899, 925)

926. **A. P. Colburn, Carl Gazley, Jr., E. M. Schoenborn, and C. S. Sutton, "Effect of local boiling and air entrainment on temperatures of liquid-cooled cylinders,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1498, Mar. 1948, pp. 1–75.

Data are presented showing the effect of local boiling on heat-transfer rates. Water, methanol, and mixtures of water and methanol were made to flow in an annular space, the outer wall of which was made of glass so as to permit the inner heated metal-liquid interface to be observed. When the wall temperature was below the boiling temperature of the fluid, the usual data for heat transfer were obtained.

However, when the wall temperature was above the fluid boiling point, even though the bulk temperature of the fluid was not above this point, an increased heat-transfer rate was obtained. The increase ranged up to a maximum of 300 per cent. The heat-transfer rates were found to be dependent upon how much the temperature of the wall was above the boiling point. The injection of air increased the heat-transfer rate up to 100 per cent.

The practical result of this phenomenon is that liquid-cooled engine cylinder walls do not overheat as badly with coolants of low boiling point as they do with less volatile coolants.

The report presents 24 pages of data. Myron Tribus, USA

927. **E. A. Farber and R. L. Scorah, "Heat transfer to water boiling under pressure,"** *Trans. Amer. Soc. mech. Engrs.*, May 1948, vol. 70, pp. 369–384.

The authors report the results of an experimental investigation of surface coefficients from wires to water boiling at various pressures. The wire was electrically heated and the heat flow calculated from the electrical input. The temperature of the wire surface was determined by means of a small thermocouple welded to it and, also, at the higher temperatures, by means of an optical pyrometer. Wires of various materials were used (nickel, tungsten, chromel A, and chromel C) and the wire diameter was 0.040 in.

The authors found that it was necessary to heat a new wire to glowing heat for about 15 min to obtain reproducible results. The wire surfaces were more uniform following the treatment.

It was found that the boiling curve (heat transfer per unit area against temperature difference) was continuous over the range of temperature differences. Different metals gave boiling curves of identical form but displaced relatively.

Increasing the gage pressure from 0 to 100 psi increased the surface coefficients at the same relative place on the curve and also decreased the temperature difference at which the maximum coefficient occurred.

The same heat transfer was obtained for three different temperature differences and with careful control it was possible to have film and nuclear boiling take place simultaneously at

different parts of the wire. Surface heat-transfer coefficients higher than 100,000 Btu per hr sq ft deg F were obtained at a gage pressure of 100 psi. No theoretical correlation of the data was attempted.

Y. S. Touloukian, USA

928. **Gustave Ribaud, "A new solution of the Fourier equation (Une solution nouvelle de l'équation de Fourier),"** *C. R. Acad. Sci. Paris*, Jan. 12, 1948, vol. 226, pp. 140–142.

The author obtains a new solution for the Fourier equation for an infinite wall in the form $\theta = t^m f(x/\sqrt{4at}) = t^m f(z)$, where θ is temperature, x is the depth, t is time, and a is the thermal diffusivity of the wall. This is a generalized form of the classical solution $\theta = f(x/\sqrt{4at})$ due to Cauchy (for which $m = 0$). Applying the author's solution to the case of an infinite wall receiving a constant heat flow, the temperature may be expressed by $\theta = \sqrt{4at}\, f(x/\sqrt{4at})$.

Ahmed D. Kafadar, USA

929. **Gustave Ribaud, "The problem of infinite wall with constant heat flow (Le problème du mur indéfini avec flux calorifique constant),"** *C. R. Acad. Sci. Paris*, Jan. 19, 1948, vol. 226, pp. 204–206.

The author determines the function f in his previous solution $\theta = t^m f(x/\sqrt{4at}) = t^m f(z)$ of the Fourier equation [*C. R. Acad. Sci. Paris*, Jan. 12, 1948, vol. 226, p. 140]. A table is given for $f(z)$, for values of z from 0 to 2. The value of $f(z)$ differs very little from e^{-2z} for values of z smaller than 0.5. •The temperature in an infinite wall receiving a constant heat flow ϕ is given by $\theta = [\phi/(\lambda\sqrt{\pi})]\sqrt{4at}\, f(z)$ where λ is thermal conductivity, a thermal diffusivity, x distance from the surface of the wall, and t is time.

It should be noted that other solutions of the Fourier equation for this case have been found (see, for instance, Carslaw and Jaeger, "Conduction of heat in solids," 1947).

Ahmed D. Kafadar, USA

930. **Gustave Ribaud, "Expansions based on a solution of the Fourier equation in the case of an infinite wall (Développements sur une solution de l'équation de Fourier dans le cas du mur d'épaisseur infinie),"** *C. R. Acad. Sci. Paris*, Feb. 9, 1948, vol. 226, pp. 449–451.

This is a discussion of the solution of the Fourier equation obtained by the author in the form

$$\theta = t^m f_m(x/\sqrt{4at}) = t^m f_m(z)$$

[*C. R. Acad. Sci. Paris*, Jan. 12, 1948, vol. 226, p. 140].

For an arbitrary m, the function f_m is defined by the differential equation

$$f_m'' + 2z f_m' - 4m f_m = 0$$

It is shown that for m positive, and for integral values of $2m$, f_m is determined by $2m$ successive differentiations. Interesting values of m discussed are $m = 1/2$ corresponding to a wall with constant heat flow, and $m = 3/2$ which corresponds to heat flow proportional to time. For m negative the solution is obtained by means of $2m$ successive integrations, and the case $m = -1/2$ corresponding to an instantaneous heat source is discussed. The case $m = 0$ corresponds to the classical solution due to Cauchy for the case in which the surface temperature of a wall is raised suddenly to a constant temperature θ_0.

Ahmed D. Kafadar, USA

931. **Marcel Devienne, "Influence of humidity on thermal conductivity of granulated materials (Influence de l'humidité sur la conduction thermique des corps granuleux),"** *C. R. Acad. Sci. Paris*, May 10, 1948, vol. 226, pp. 1512–1513.

The author briefly discusses experimental work on the apparent thermal conductivity of granulated materials as influenced by moisture. Tests on steel, glass, and lead spheres 3.3 mm in diam showed the ratio of the apparent thermal conductivities for two extreme conditions when the spaces were filled with water and dry air to be 3.72. The author points out that the influence of moisture on the apparent thermal conductivity of granulated solids is most effective when condensation takes place at the points of contact between the spheres.

Y. S. Touloukian, USA

932. **R. Kronig and N. Schwarz, "On the theory of heat transfer from a wire in an electric field,"** *Appl. sci. Res. Sec. A*, 1947, vol. 1, no. 1, pp. 35–46.

If an electric field is applied between a horizontal wire and a concentric cylinder filled with certain gases, the heat transfer from the wire is increased, due to the electrostrictive forces which modify the flow pattern corresponding to free convection. For para-electric gases, an increase in heat transfer up to 50 per cent was observed by Senftleben and Braun. The effect was very much smaller for dielectric gases. The phenomenon offers possibilities for the continuous analysis of gas mixtures containing a para-electric component.

An analysis of this phenomenon is presented, utilizing dynamical similarity, which indicates that for wires in an electric field

$$Nu = f_1(Gr \cdot Pr) + f_2(El \cdot Pr)$$

where Nu, Gr, Pr are the Nusselt, Grashof, and Prandtl moduli and El is a new modulus analogous to the Grashof modulus, in which the gravitational force per unit mass g is replaced by the term $\gamma E^2/d$. In the latter expression γ is the temperature coefficient of electric susceptibility, E the electric field strength, and d the diameter of the wire.

A plot of $Nu - f_1(Gr \cdot Pr)$ against $(El \cdot Pr)$, calculated from the data of Senftleben and Braun on a 0.03-mm-diam platinum wire surrounded by argon, oxygen, and ethyl chloride, indicates excellent correlation between the analysis and experiment.

R. C. Martinelli, USA